MEAN-VARIANCE PORTFOLIO OPTIMIZATION ON ISLAMIC STOCKS THAT THERE CAUSALITY RELATIONSHIP WITH MARKET INDEX

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Abstract: In this paper, we conduct an analysis of mean-variance portfolio optimization on Islamic stocks that has a causal relationship with the market index. It is assumed that the return of Islamic stocks that analyzed, there are causal relationship with the market index return. To determine the causal relationship is done by using the Granger causality test. When there are Islamic stock returns does not have a causal relationship with the market index, mean estimated using the models of autoregressive moving average (ARMA). While for the Islamic stocks that have a causal relationship with the market index, mean is estimated using vector autoregressive (VAR). Furthermore, to estimate the volatility we have done by using the models of generalized autoregressive conditional heteroscedastic (GARCH). Based on estimator of ARMA, VAR, and GARCH models, predictions one period ahead is done determines the values of mean and variance. The values of the mean and variance are then used for portfolio optimization process. Portfolio optimization is done by using Mean-Variance model approach. The optimization is used to determine the weight of investment funds allocated. So the results can be taken into consideration for investors, to determine the percentage allocation of funds invested in each of the Islamic stocks.

Keywords: Granger Causality, ARMA, VAR, GARCH, Mean-Variance portfolio optimization.

1. Introduction

Capital market in Indonesia has begun to develop into the Islamic capital market since July 3, 1997, PT. Denareksa Investment Management launched (Islamic Mutual Funds). Indonesia Stock Exchange in cooperation with PT. Denareksa Investment Management launched the Jakarta Islamic Index (JII) on 3 July 2000 the Islamic capital market is intended to guide investors who want to invest their funds in Islamic. Of course, the stock traded in the Islamic capital market are shares that do not conflict with the provisions of Islam (Aziz and Kurniawan, 2007).

In the theory of investment, each stock will result in the return and risk. Return the results obtained from the investment or level of benefits enjoyed by investors, on an investment that does (Aziz & Kurniawan, 2007; Chapakia & Sanrego, 2007). Formation stock price is due to the demand and supply of the stock. Words another stock prices are formed by supply and demand of the stock. Supply and demand that occur due to many factors, both specific to the nature of the stock and the macro factors like interest rates, inflation, exchange rates and non-economic factors such as the social and political condition, and other factors (Aziz and Kurniawan, 2007).

Stocks are securities that have a high degree of risk. Risk or loss cannot be eliminated in an investment, and the risk can be seen from the high and low levels of fluctuations (volatility) of stock prices. Therefore, to determine the level of risk-factor relationships to know what factors influence it, and how big influence. Macro variables such as interest rates and exchange rates always affect the systematic risk in any investment, especially investment in each stock, either common stock or Islamic
stock (Chapakia & Sanrego, 2007; Constantinou et al., 2004). Risk cannot be eliminated, but can be minimized by setting up an investment portfolio. Establishment of an investment portfolio is essentially allocated capital in a few selected stocks, or often referred to diversify investments (Panjer et al., 1998). The purpose of the establishment of the investment portfolio is to get a certain return with minimum risk levels, or to get maximum returns with limited risk. To achieve reviews these objectives, the investor is deemed necessary to conduct analysis of optimal portfolio selection. Analysis of portfolio selection can be done with optimum investment portfolio optimization techniques (Goto & Yan Xu, 2012).

Therefore, this paper studied the portfolio optimization models of Mean-Variance, with assumed that the volatility of Islamic stock price returns analyzed have a causal relationship with the composite stock price index is estimated by a generalized autoregressive conditional heteroscedastic (GARCH) model approach.

2. Methodology

The method of analysis in this paper stated in the following stages:

**Data transformation.** Suppose \( X_t \) closing value Composite Stock Price Index at the time \( t \) where \( t = 1, 2, ..., T \) with \( T \) the number of data observations. Suppose also \( X_t \) IHSG closing returns at the time \( t \), the return value can be calculated by the equation (Tsay, 2005):

\[
X_t = \ln \left( \frac{X_t}{X_{t-1}} \right)
\]  

(1)

**Stationary test data.** Stationary test data is done with the unit root test to see if the time series data used stationary or not. Stationary test data in this study using the Augmented Dickey Fuller (ADF) test. This test by comparing the value ADF test with Mackinnon Critical Value 1%, 5%, 10% by the following equation (Gujarati, 2003: 817):

\[
\Delta Z_t = \alpha_0 + \beta t + \lambda Z_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Z_{t-i} + \epsilon_t
\]

where \( \alpha_0 \) is a deterministic function of time index \( t \) and \( \Delta Z_t = Z_t - Z_{t-1} \) is the first difference. In practice \( \alpha_0 \) can be zero or constant. t-ratio of \( \hat{\lambda} \) is

\[
\text{ADF - test} = \frac{\hat{\lambda}}{\text{std}(\hat{\lambda})}
\]

(3)

where \( \hat{\lambda} \) is estimator of \( \lambda \), which is referred to as the Augmented Dickey Fuller at the unit root test.

The hypothesis used in this study is \( H_0 : \lambda = 0 \) time series data contain a unit root, and \( H_0 : \lambda < 0 \) time series data does not contain a unit root. If \( ADF_{\text{test}} > \text{MacKinnon critical value} \), then reject \( H_0 \). Whereas if \( ADF_{\text{test}} < \text{MacKinnon critical value} \), then receive \( H_0 \) (Shi-Jie Deng, 2004; Tsay, 2005).

**Determination of the optimal lag.** An important step in using the VAR model is to determine the optimal lag is included in the model. If the lag is used in the model is too little then the residuals of the regression will not show the white noise so that the model can not accurately estimate the actual error. However, if the lag is entered too much, it can reduce the ability to resist

\( H_0 \) because too many additional parameters that will reduce degree of freedom (Gujarati, 2003:853).

To assess the quality of a model can be seen from the value of the Akaike Information Criterion (AIC) or Swarshartz Information Criterion (SIC) (Gujarati, 2003:537).

a. Akaike Information Criterion (AIC)

\[
\text{AIC} = e^{2k/n} \sum \hat{\epsilon}_i^2
\]

\[
= e^{2k/n} \frac{RSS}{n}
\]

(4)
where $k$ is the number of explanatory variables plus a constant and $n$ is the number of data.

b. **Swachartz Information Criterion (SIC)**

\[
\text{SIC} = n^{k/n} \sum \bar{u}_i^2 = n^{k/n} \frac{\text{RSS}}{n}
\]

where $k$ is the number of the explanatory variables plus a constant and $n$ is the number of data.

Determination of the optimal lag using (information criterion) is obtained by selecting the criteria that have the smallest value among the various proposed lag.

**Estimation of VAR models.** VAR model is the development of models of autoregressive (AR).

If at the AR observation time is now affected by previous observations in that location, then at the VAR model of the observation time is now affected by previous observations at these locations and other locations. VAR model of order $p$ is denoted by $\text{VAR}(p)$ expressed in the following equation (Tsay, 2005):

\[
Z_t = \phi_1 Z_{t-1} + \cdots + \phi_p Z_{t-p} + \alpha_t
\]

Where $Z_t$ is a vector of size $nx1$, containing $n$ variables in VAR at the time $t$, $\phi_p$ is a matrix of size of $nxn$ containing the coefficients in the VAR and $\alpha_t$ is a vector of size of $nx1$ containing the residuals of the VAR model. So for the bivariate vector autoregressive models ($n=2$), can be written:

\[
\begin{bmatrix}
Z_{t,1} \\
Z_{t,2}
\end{bmatrix} =
\begin{bmatrix}
\phi_{1,1} & \phi_{1,2} \\
\phi_{2,1} & \phi_{2,2}
\end{bmatrix}
\begin{bmatrix}
Z_{t-1,1} \\
Z_{t-1,2}
\end{bmatrix} + \cdots +
\begin{bmatrix}
\phi_{1,p} & \phi_{1,2p} \\
\phi_{2,p} & \phi_{2,2p}
\end{bmatrix}
\begin{bmatrix}
Z_{t-p,1} \\
Z_{t-p,2}
\end{bmatrix} +
\begin{bmatrix}
\alpha_{1,t} \\
\alpha_{2,t}
\end{bmatrix}
\]

**Diag nostic Test.** One important step in estimating a model is to test whether the model has been estimated that it deserves to be used or not. This feasibility testing or diagnostic tests, that test for the presence of serial correlation between the residuals at the same lag. In this study, a diagnostic test used is the Portmanteau test, the test statistic as follows (Tsay, 2005; Sifriyani, 2004; Wo’zniak, 2009):

\[
Q = n(n+2) \sum_{k=1}^{m} \frac{\bar{r}_k^2}{n-k}
\]

Distributed $\chi^2_{m-p}$ with $\bar{r}_k$ is the residual correlation, $m$ is the number of lagged residuals, $p$ is the order of the VAR model, and $n$ is the number of residuals. The hypothesis used at the this stage is $H_0$: Does not residual serial autocorrelation, and $H_1$: Residual serial autocorrelation. If the $Q > \chi^2_{m-p}$ then reject $H_0$. Akan tetapi jika $Q < \chi^2_{m-p}$ maka terima $H_0$.

**Granger Causality Test.** Granger causality test is used to see the relationship of a variable to another variable. $Z_2$ called Granger cause $Z_1$ if the past values of the variable $Z_2$ may helpful to explain the variable $Z_1$ (Tsay, 2005; Febrian & Herawan, 2009; Wo’zniak, 2009). Granger approach tries to answer whether the IHSG ($X_1$) affect the price of Islamic stock ($P_\alpha$) and vice versa. In general, to test the Granger causality using two common forms of the VAR model, ie:

\[
X_t = \sum_{j=1}^{q} \alpha_j P_{t-j} + \sum_{j=1}^{q} \beta_j X_{t-j} + \epsilon_{1t}
\]

\[
P_\alpha = \sum_{j=1}^{q} \epsilon_j P_{\alpha-t-j} + \sum_{j=1}^{q} d_j X_{t-j} + \epsilon_{2t}
\]

It is assumed that $\epsilon_{1t}, \epsilon_{2t}$ not correlate. From the results of the regression equation (9) and (10) above, it will produce four possible values of the regression coefficients, each coefficient is:
1) If \( \sum_{j=1}^{q} \alpha_j \neq 0 \) and \( \sum_{j=1}^{q} d_j = 0 \), then there is a one-way causality Islamic stock price to IHSG.

2) If \( \sum_{j=1}^{q} \alpha_j = 0 \) and \( \sum_{j=1}^{q} d_j \neq 0 \), then there is a one-way causality IHSG against Islamic stock price.

3) If \( \sum_{j=1}^{q} \alpha_j = 0 \) and \( \sum_{j=1}^{q} d_j = 0 \), then there is no causality between IHSG against Islamic stock price as well as between Islamic stock price to IHSG.

4) If \( \sum_{j=1}^{q} \alpha_j \neq 0 \) and \( \sum_{j=1}^{q} d_j \neq 0 \), then there is a bidirectional causality between IHSG against Islamic stock price and between (Islamic stock quotes) against IHSG.

To test whether a variable \( P_r \) influence \( X_t \) is to find the Residual Sum of Square of unrestricted regression equation, namely \( RSS_{UR} \) and the restricted regression, ie \( RSS_R \), then tested using the \( F \) : test

\[
F = \left(\frac{n-k}{s}\right) \frac{RSS_R - RSS_{UR}}{RSS_R}
\]  

where \( n \) is the number of data observations, \( k \) is the number parameters of the unrestricted equation, and \( s \) a number of parameters of the equation restricted, with hypothesis as follows: \( H_0 \): \( P_r \) not influence \( X_t \), and \( H_1 \): \( P_r \) influence \( X_t \). The rule, if \( F_{stat} > F_{label} \) then reject \( H_0 \), it can be said that \( P_r \) influence \( X_t \). If \( F_{stat} < F_{label} \) then receive \( H_0 \), so it can be said that \( P_r \) not influence \( X_t \). The same way we done for the case, if \( X_t \) influence \( P_r \) (Tsay, 2005; Febrian & Herawany, 2009).

**Volatility Models.** Volatility models in time series data in general can be analyzed using GARCH models. Suppose \( \{r_{it}\} \) is Islamic stock returns \( i \) at time \( t \) is stationary, the residuals of the mean model for Islamic stock \( i \) at time \( t \) is \( a_{it} = r_{it} - \mu_{it} \). The residual sequence \( \{a_{it}\} \) follows the model GARCH(\( g, s \)) when for each has the following equation:

\[
a_{it} = \sigma _{it} \varepsilon _{it}, \quad \sigma _{it}^2 = \alpha _{i0} + \sum _{k=1}^{g} \alpha _{ik} a_{it-k}^2 + \sum _{j=1}^{s} \beta _{ij} \sigma _{it-j}^2 + \varepsilon _{it},
\]  

with \( \{\varepsilon _{it}\} \) is a sequence of residual volatility models, namely the sequence of random variables are independent and identically distributed (IID) with mean 0 and variance 1. Parameter coefficients satisfy the property that \( \alpha _{i0} > 0, \alpha _{ik} \geq 0, \beta _{ij} \geq 0, \) and \( \sum _{k=1}^{\max (g,s)} \alpha _{ik} + \beta _{ij} < 1 \) (Shi-Jie Deng, 2004; Tsay, 2005).

Volatility modeling process steps include: (i) The estimated mean model, (ii) Test of ARCH effects, (iii) Identification of the model, (iv) The estimated volatility models, (v) Test of diagnosis, and (vi) Prediction (Tsay, 2005).
**Prediction of 1-Step Ahead.** Using the mean and volatility models, aiming to calculate the prediction of mean \( \hat{\mu}_t = \hat{\mu}_h(l) \) and volatility \( \hat{\sigma}^2_{it} = \hat{\sigma}^2_{ih}(l) \), for \( l \)-period ahead of the starting point prediction \( h \) (Tsay, 2005; Febrian & Herwany, 2009). The prediction results of mean \( \hat{\mu}_t = \hat{\mu}_h(l) \) and volatility \( \hat{\sigma}^2_{it} = \hat{\sigma}^2_{ih}(l) \), will then be used for portfolio optimization calculations below.

**Optimisasi portfolio.** Suppose \( r_{it} \) Islamic stock return \( i \) at time \( t \), where \( i = 1, \ldots, N \) with \( N \) the number of stocks that were analyzed, and \( t = 1, \ldots, T \) with \( T \) the number of Islamic stock price data observed. Suppose also \( w' = (w_1, \ldots, w_N) \) weight vector, \( r' = (r_{1t}, \ldots, r_{Nt}) \) vector stock returns, and \( e' = (1, \ldots, 1) \) unit vector. Portfolio return can be expressed as Error! Bookmark not defined. with \( w'e = 1 \) (Sukono et al., 2011; Panjer et al., 1998). Suppose \( \mu' = (\mu_{1t}, \ldots, \mu_{Nt}) \), expectations of portfolio \( \mu_p \) can be expressed as:

\[
\mu_p = E[r_p] = w'\mu .
\]

Suppose given covariance matrix \( \Sigma = (\sigma_{ij})_{i,j = 1, \ldots, N} \), where \( \sigma_{ij} = Cov(r_{it}, r_{jt}) \). Variance of the portfolio return can be expressed as follows:

\[
\sigma_p^2 = w'\Sigma w .
\]

**Definition 1. (Panjer et al., 1998).** A portfolio \( p^* \) called (Mean-variance) efficient if there is no portfolio \( p \) with \( \mu_p \geq \mu_{p^*} \) and \( \sigma_p^2 < \sigma_{p^*}^2 \) (Panjer et al., 1998).

To get efficient portfolio, typically using an objective function to maximize

\[
2\tau\mu_p - \sigma_p^2 , \tau \geq 0
\]

where the parameters of the investor's risk tolerance. Means, for investors with risk tolerance \( \tau \) (\( \tau \geq 0 \)) need to resolve the problem of portfolio

\[
\text{Maximize} \{2rw'\mu - w'\Sigma w\}
\]

(15)

the condition \( w'e = 1 \)

Please note that the completion of (6), for all \( \tau \in [0, \infty) \) form a complete set of efficient portfolios. Set of all points in the diagram- \((\mu_p, \sigma_p^2)\) related to efficient portfolio so-called surface efficient (efficient frontier) (Goto & Yan Xu, 2012; Sukono et al., 2011).

Equation (6) is the optimization problem of quadratic convex (Panjer et al., 1998). Lagrangian multiplier function of the portfolio optimization problem is given by

\[
L(w, \lambda) = 2rw'\mu - w'\Sigma w + \lambda(w'e - 1) .
\]

(16)

Based on the Kuhn-Tucker theorem, the optimality condition of equation (7) is \( \frac{\partial L}{\partial w} = 0 \) and \( \frac{\partial L}{\partial \lambda} = 0 \).

Completed two conditions of optimality mentioned equation, the equation would be the optimal portfolio weights as follows:

\[
w^* = \frac{1}{e'\Sigma^{-1}e}\Sigma^{-1}e + \tau(\Sigma^{-1}\mu - \frac{e'\Sigma^{-1}\mu}{e'\Sigma^{-1}e}\Sigma^{-1}e) .
\]

(17)

Furthermore, with substituting \( w^* \) into equation (6) and (7), respectively obtained the values of the expectation and variance of the portfolio (Sukono et al., 2011). As a numerical illustration, will be analyzed some Islamic stocks as the following.
3. Results and Discussion

3.1 Data an Analyzed
The research object in this study is some stock price sharia (ASII, AALI, KLBF, LSIP, and PTBA) and the stock price index (IHSG). The data used in this study is a secondary data that is both time series (time series) daily. Islamic stock price data used is the closing stock price of sharia in the capital market for 262 days starting with the March 29, 2013 till April 3, 2014 were obtained by the www.finance.yahoo.com and data used IHSG IHSG is the data for 262 days starting with the March 5, 2013 till March 28, 2014 were obtained of the www.bi.go.id.

3.2 Data analysis
As explained in section 2, that the methodology of the analysis carried stages as follows:

**Calculation Islamic Stock Returns and IHSG Return.** Calculation of Islamic stock returns follow equation (1). The results of the calculation of the Islamic stock returns ASII which is one of the stock prices used in this study are as follows:

\[ p_1 = \ln \left( \frac{7395}{7295} \right) = 0.013615 \]

Calculations were continued until the last date.

Stationary Test Results of the Data. Suppose \( p_{1t} \) is the Islamic stock price return of ASII, \( p_{2t} \) is the Islamic stock price return of AALI, \( p_{3t} \) is the Islamic stock price return of KLBF, \( p_{4t} \) is the Islamic stock price return of LSIP, and \( p_{5t} \) is the Islamic stock price return of PTBA. The hypothesis used is \( H_0 : \text{ADF}_{t} > \text{MacKinnon Critical Value} \) (there is a unit root), and \( H_1 : \text{ADF}_{t} < \text{MacKinnon Critical Value} \) (there is no unit root).

Stationarity test was done according to equation (3). The results of the unit root tests using EViews 5 in this study are presented in Table 1 as follows:

<table>
<thead>
<tr>
<th>Variables</th>
<th><strong>Critical Value at level:</strong></th>
<th>ADF&lt;sub&gt;test&lt;/sub&gt;</th>
<th>Prob.</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_t )</td>
<td>-3.9937</td>
<td>3.4272</td>
<td>-3.1368</td>
<td>-18.8207</td>
</tr>
<tr>
<td>( p_{1t} )</td>
<td>-3.9936</td>
<td>3.4271</td>
<td>-3.1369</td>
<td>-14.8068</td>
</tr>
<tr>
<td>( p_{2t} )</td>
<td>-3.9937</td>
<td>3.4272</td>
<td>-3.1369</td>
<td>-14.9098</td>
</tr>
<tr>
<td>( p_{3t} )</td>
<td>-3.9937</td>
<td>3.4272</td>
<td>-3.1369</td>
<td>-16.38253</td>
</tr>
<tr>
<td>( p_{4t} )</td>
<td>-3.9937</td>
<td>3.4272</td>
<td>-3.1369</td>
<td>-14.64901</td>
</tr>
</tbody>
</table>

Table 1. Stationary Test Results of the Data
Estimation results VAR model using the Akaike Information Criterion (4), by using statistical software EVViews 5 based optimal lag is selected as the order for each variable.

Selection of Optimal Lag Results. Estimation results VAR model using the Akaike Information Criteria (AIC) and Swachartz Information Criteria (SIC) refers to equation (4), by using statistical software EVViews 5, the selection of lag length obtained are presented in Table 2 as follows.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Criteria</th>
<th>lag</th>
<th>Criteria Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$ and $p_{1t}$</td>
<td>AIC</td>
<td>1</td>
<td>-13.62957*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.58249</td>
<td></td>
</tr>
<tr>
<td>$x_t$ and $p_{2t}$</td>
<td>AIC</td>
<td>2</td>
<td>-13.85317*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.78568</td>
<td></td>
</tr>
<tr>
<td>$x_t$ and $p_{3t}$</td>
<td>AIC</td>
<td>1</td>
<td>-13.99583*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.95771</td>
<td></td>
</tr>
<tr>
<td>$x_t$ and $p_{4t}$</td>
<td>AIC</td>
<td>1</td>
<td>-13.51475*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.47074</td>
<td></td>
</tr>
<tr>
<td>$x_t$ and $p_{5t}$</td>
<td>AIC</td>
<td>3</td>
<td>-13.46223*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.39919</td>
<td></td>
</tr>
<tr>
<td>$x_t$ and $p_{6t}$</td>
<td>AIC</td>
<td>2</td>
<td>-14.00986*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.96308</td>
<td></td>
</tr>
</tbody>
</table>

From Table 2 seen that the two criteria provide information different maximum lag length for each variable. However, the maximum lag selected for each variable is the lag generated by the criteria of AIC, because the AIC criterion always gives the smallest value for each variable.

Estimation of VAR Models. Estimation of Vector Autoregressive models (VAR) refers to equation (5) by using the software EVViews 5 based optimal lag is selected as the order for each variable are as follows:

1. Estimate the model for variable $x_t$ and $p_{1t}$ is VAR (1) as follows:
   
   $x_t = 0.000260 - 0.156796x_{t-1} + 0.003615p_{u-1} + \varepsilon_1$
   
   $p_{1t} = 0.0015686 - 0.959334x_{t-1} + 0.077881p_{u-1} + \varepsilon_2$

2. Estimate the model for variable $x_t$ and $p_{2t}$ is VAR (2) as follows:
   
   $x_t = 0.000269 - 0.181777x_{t-1} - 0.113466x_{t-2} - 0.005654p_{2t-1} - 0.014637p_{2t-2} + \varepsilon_1$
   
   $p_{2t} = -0.001367 + 0.640545x_{t-1} + 1.202416x_{t-2} + 0.054533p_{2t-1} - 0.021550p_{2t-2} + \varepsilon_2$

3. Estimate the model for variable $x_t$ and $p_{3t}$ is VAR (1) as follows:
   
   $x_t = 0.000304 - 0.145612x_{t-1} - 0.019307p_{3t-1} + \varepsilon_1$
   
   $p_{3t} = 0.002274 + 0.051171x_{t-1} - 0.022476p_{3t-1} + \varepsilon_2$

4. Estimate the model for variable $x_t$ and $p_{4t}$ is VAR (1) as follows:
   
   $x_t = 0.000247 - 0.152360x_{t-1} - 0.010134p_{4t-1} + \varepsilon_1$
   
   $p_{4t} = -0.001549 + 0.55630x_{t-1} + 0.086274p_{4t-1} + \varepsilon_2$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Criteria</th>
<th>lag</th>
<th>Criteria Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$ and $p_{5t}$</td>
<td>AIC</td>
<td>2</td>
<td>-13.62957*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.58249</td>
<td></td>
</tr>
<tr>
<td>$x_t$ and $p_{6t}$</td>
<td>AIC</td>
<td>1</td>
<td>-13.85317*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.78568</td>
<td></td>
</tr>
<tr>
<td>$x_t$ and $p_{7t}$</td>
<td>AIC</td>
<td>1</td>
<td>-13.99583*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.95771</td>
<td></td>
</tr>
<tr>
<td>$x_t$ and $p_{8t}$</td>
<td>AIC</td>
<td>3</td>
<td>-13.46223*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.39919</td>
<td></td>
</tr>
<tr>
<td>$x_t$ and $p_{9t}$</td>
<td>AIC</td>
<td>2</td>
<td>-14.00986*</td>
</tr>
<tr>
<td>SIC</td>
<td>0</td>
<td>-13.96308</td>
<td></td>
</tr>
</tbody>
</table>
(5) Estimate the model for variable $x_t$ and $p_{5t}$ is VAR (3) as follows:

$$x_t = 0.000307 - 0.185075x_{t-1} - 0.149012x_{t-2} - 0.119515x_{t-3} - 0.004608p_{5t-1}$$

$$+ 0.004667p_{5t-2} - 0.021095p_{5t-3} + \varepsilon_1$$

$$p_{5t} = -0.001575 + 1.25947x_{t-1} + 0.843088x_{t-2} - 0.001575x_{t-3} + 0.031861p_{5t-1}$$

$$+ 0.008715p_{5t-2} - 0.025051p_{5t-3} + \varepsilon_2$$

**Diagnostic Test Results.** To find the model estimates that have been made unfit for use, then the diagnostic test, in this study used diagnostic test is the Portmanteau test. The hypothesis used in this test is as follows $H_0$: There is no serial correlation of the residuals, and $H_1$: Serial correlation of the residuals. Diagnostics test are done according to the equation (8). Diagnostic Test Results Portmanteau using software EViews 5 can be seen in Table 3 below.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$x_t$ and $p_{1t}$</td>
<td>Q-stat</td>
</tr>
<tr>
<td>Prob.</td>
<td>NA</td>
</tr>
<tr>
<td>$x_t$ and $p_{2t}$</td>
<td>Q-stat</td>
</tr>
<tr>
<td>Prob.</td>
<td>NA</td>
</tr>
<tr>
<td>$x_t$ and $p_{3t}$</td>
<td>Q-stat</td>
</tr>
<tr>
<td>Prob.</td>
<td>NA</td>
</tr>
<tr>
<td>$x_t$ and $p_{4t}$</td>
<td>Q-stat</td>
</tr>
<tr>
<td>Prob.</td>
<td>NA</td>
</tr>
<tr>
<td>$x_t$ and $p_{5t}$</td>
<td>Q-stat</td>
</tr>
<tr>
<td>Prob.</td>
<td>NA</td>
</tr>
<tr>
<td>$x_t$ and $p_{6t}$</td>
<td>Q-stat</td>
</tr>
<tr>
<td>Prob.</td>
<td>NA</td>
</tr>
</tbody>
</table>

Based on Table 3 probability for Q-stat on all variables collected in mind that models the null hypothesis is not rejected at the 5% significance level, meaning that there is no serial correlation between the two variables. It can be concluded that all the estimated model is the best model for each variable.

**Results of Granger Causality Test.** Seen in causality test causal relationship between IHSG with some Islamic stock price. Granger causality test was done according to equation (11). Causality test results using the software EViews 5 can be seen of the probability value of the $F$ test. The hypothesis used in this study are as follows $H_0$: Causality does not occur, and $H_1$: Causality occurs. Causality test results are presented in Table 4 below.

<table>
<thead>
<tr>
<th>Variables</th>
<th>$H_0$</th>
<th>$F$-statistik</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$ does not Granger Cause $p_{1t}$</td>
<td>$F_{3,18}$</td>
<td>4.44043</td>
<td>0.03682</td>
</tr>
<tr>
<td>$p_{1t}$ does not Granger Cause $x_t$</td>
<td>$F_{1,20}$</td>
<td>0.18754</td>
<td>0.66534</td>
</tr>
<tr>
<td>$x_t$ does not Granger Cause $p_{2t}$</td>
<td>$F_{3,18}$</td>
<td>4.77611</td>
<td>0.00920</td>
</tr>
<tr>
<td>$p_{2t}$ does not Granger Cause $x_t$</td>
<td>$F_{1,20}$</td>
<td>1.52752</td>
<td>0.21906</td>
</tr>
<tr>
<td>$x_t$ does not Granger Cause $p_{3t}$</td>
<td>$F_{3,18}$</td>
<td>0.01765</td>
<td>0.89443</td>
</tr>
</tbody>
</table>

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Based on Table 4 seen that the variable $x_i$ influence the variable $p_{li}$ because the probability is less than the significance level of 5% so $H_0$ rejected. While the, $p_{li}$ does not affect the variable $x_i$ because the probability of more than 5% significance level that $H_0$ accepted.

Variable $x_i$ influence the variable $p_{3i}$ because the probability is less than the significance level of 5% so $H_0$ rejected. While the, $p_{3i}$ does not affect the variable $x_i$ because the probability of more than 5% significance level that $H_0$ accepted.

The variable $x_i$ does not affect the variable $p_{4i}$ because the probability of more than 5% significance level that $H_0$ accepted. Likewise $p_{4i}$ does not affect the variable $x_i$ because the probability of more than 5% significance level that $H_0$ accepted.

The variable $x_i$ does not affect the variable $p_{5i}$ because the probability of more than 5% significance level that $H_0$ accepted. Likewise with $p_{4i}$ does not affect the variable $x_i$ because the probability of more than 5% significance level that $H_0$ accepted.

Variable $x_i$ influence the variable $p_{5i}$ because the probability is less than the significance level of 5% so $H_0$ rejected. Likewise with $p_{5i}$ influence the variable $x_i$ because the probability is less than the significance level of 5% so $H_0$ rejected.

**Estimation of Volatility Models.** From the estimation results of the VAR model (1), (2), (3), (4), and (5) above, especially for the estimator equation $p_{li}$, $p_{2i}$, $p_{3i}$, $p_{4i}$, and $p_{5i}$, then used to estimate volatility models. Volatility model estimation is done by using a GARCH model. Estimation is done with reference to equation (12), and with the help of software Eviews 8, and the results are as follows:

1) Islamic stock ASII follow the model GARCH(1,1) with equation:
   \[ \sigma_{li}^2 = 0.000012 + 0.040017\epsilon_{li-1}^2 + 0.930451\varepsilon_{li-1}^2 + u_{li} \]

2) Islamic stock AALI follow the model GARCH(1,1) with equation:
   \[ \sigma_{2i}^2 = 0.000053 + 0.234048\epsilon_{2i-1}^2 + 0.921716\varepsilon_{2i-1}^2 + u_{2i} \]

3) Islamic stock KLBF follow the model GARCH(1,1) with equation:
   \[ \sigma_{3i}^2 = 0.000021 + 0.056124\epsilon_{3i-1}^2 + 0.933208\varepsilon_{3i-1}^2 + u_{3i} \]

4) Islamic stock LSIP follow the model GARCH(1,1) with equation:
   \[ \sigma_{4i}^2 = 0.000019 + 0.054352\epsilon_{4i-1}^2 + 0.954700\varepsilon_{4i-1}^2 + u_{4i} \]

5) Islamic stock PTBA follow the model GARCH(1,1) with equation:
   \[ \sigma_{5i}^2 = 0.000018 + 0.239176\epsilon_{5i-1}^2 + 0.924640\varepsilon_{5i-1}^2 + u_{5i} \]

Based on the ARCH-LM test statistics, the residuals of the models for Islamic stock ASII, AALI, KLBF, LSIP, dan PTBA there is no element of ARCH, and also has white noise. Mean and volatility models are then used to calculate the values \( \hat{\mu}_{ii} = \hat{\mu}_{il}(l) \) and \( \hat{\sigma}_{ii}^2 = \sigma_{il}^2(1) \), \( i = 1,...,5 \), recursively.

**Prediction of Mean and Variance Values.** Models of mean and volatility estimation results of five Islamic stocks in the previous stage, is then used to perform one step ahead prediction for the mean and variance values. The results predicted mean and volatility values are given in Table-1 below.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{3i}$ does not Granger Cause $x_i$</td>
<td>3.76044</td>
<td>0.05357</td>
</tr>
<tr>
<td>$x_i$ does not Granger Cause $p_{4i}$</td>
<td>1.30759</td>
<td>0.25390</td>
</tr>
<tr>
<td>$p_{4i}$ does not Granger Cause $x_i$</td>
<td>1.67728</td>
<td>0.19645</td>
</tr>
<tr>
<td>$x_i$ does not Granger Cause $p_{5i}$</td>
<td>2.65245</td>
<td>0.04923</td>
</tr>
<tr>
<td>$p_{5i}$ does not Granger Cause $x_i$</td>
<td>2.80617</td>
<td>0.04026</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
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<td>1.30759</td>
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</tr>
<tr>
<td>$p_{4i}$ does not Granger Cause $x_i$</td>
<td>1.67728</td>
<td>0.19645</td>
</tr>
<tr>
<td>$x_i$ does not Granger Cause $p_{5i}$</td>
<td>2.65245</td>
<td>0.04923</td>
</tr>
<tr>
<td>$p_{5i}$ does not Granger Cause $x_i$</td>
<td>2.80617</td>
<td>0.04026</td>
</tr>
</tbody>
</table>

**Table 5. Predictive Values of Mean and Variance One Period Ahead For Each Islamic Stocks**

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**Portfolio Optimization.** Optimasasi portfolio is done with the aid of software Matlab-7. Because five stocks used for portfolio formation, it is determined that the vector \( e^T = (1 1 1 1 1) \). From the Table 5 can be structured vector average \( \mu^T = (0.000102 0.001740 0.000501 0.002979 0.000168) \). Assuming the covariance between stock \( \sigma_{ij} = 0 \), from the Table 5 covariance matrix can be formed by rounding to four decimal as

\[
\Sigma = \begin{bmatrix}
0.0001 & 0 & 0 & 0 & 0 \\
0 & 0.0014 & 0 & 0 & 0 \\
0 & 0 & 0.0003 & 0 & 0 \\
0 & 0 & 0 & 0.0015 & 0 \\
0 & 0 & 0 & 0 & 0.0003 \\
\end{bmatrix}
\]

\[
\Sigma^{-1} = \begin{bmatrix}
8333.3 & 0 & 0 & 0 & 0 \\
0 & 699.30 & 0 & 0 & 0 \\
0 & 0 & 4000.0 & 0 & 0 \\
0 & 0 & 0 & 675.70 & 0 \\
0 & 0 & 0 & 0 & 4000.0 \\
\end{bmatrix}
\]

Furthermore, some of the values determined risk tolerance \( \tau \) viable, here taken \( \tau = 0,...,9 \). Then the portfolio weights are calculated using the equation (17), while the average portfolio is calculated using equation (13), and the variance as a risk measure is calculated using equation (14). The results are given in Table 6 below.

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( w_3 )</th>
<th>( w_4 )</th>
<th>( w_5 )</th>
<th>( \hat{\mu}_w )</th>
<th>( \hat{\sigma}^2_w )</th>
<th>( \hat{\mu}_w / \hat{\sigma}^2_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4714</td>
<td>0.0392</td>
<td>0.2257</td>
<td>0.0376</td>
<td>0.2262</td>
<td>0.00038486</td>
<td>0.01200</td>
<td>0.032072</td>
</tr>
<tr>
<td>1</td>
<td>0.4714</td>
<td>0.0392</td>
<td>0.2257</td>
<td>0.0376</td>
<td>0.2262</td>
<td>0.00038579</td>
<td>0.01201</td>
<td>0.032122</td>
</tr>
<tr>
<td>2</td>
<td>0.4714</td>
<td>0.0391</td>
<td>0.2257</td>
<td>0.0376</td>
<td>0.2262</td>
<td>0.00038666</td>
<td>0.01202</td>
<td>0.032168</td>
</tr>
<tr>
<td>3</td>
<td>0.4714</td>
<td>0.0391</td>
<td>0.2257</td>
<td>0.0375</td>
<td>0.2262</td>
<td>0.00038714</td>
<td>0.01204</td>
<td>0.032154</td>
</tr>
<tr>
<td>4</td>
<td>0.4715</td>
<td>0.0391</td>
<td>0.2257</td>
<td>0.0375</td>
<td>0.2262</td>
<td>0.00038743</td>
<td>0.01207</td>
<td>0.032099</td>
</tr>
<tr>
<td>5</td>
<td>0.4715</td>
<td>0.0391</td>
<td>0.2257</td>
<td>0.0375</td>
<td>0.2263</td>
<td>0.00038761</td>
<td>0.01210</td>
<td>0.032034</td>
</tr>
<tr>
<td>6</td>
<td>0.4716</td>
<td>0.0390</td>
<td>0.2257</td>
<td>0.0374</td>
<td>0.2263</td>
<td>0.00038774</td>
<td>0.01214</td>
<td>0.031939</td>
</tr>
<tr>
<td>7</td>
<td>0.4718</td>
<td>0.0390</td>
<td>0.2256</td>
<td>0.0373</td>
<td>0.2264</td>
<td>0.00038782</td>
<td>0.01219</td>
<td>0.031815</td>
</tr>
<tr>
<td>8</td>
<td>0.4721</td>
<td>0.0388</td>
<td>0.2256</td>
<td>0.0370</td>
<td>0.2265</td>
<td>0.00038787</td>
<td>0.01225</td>
<td>0.031663</td>
</tr>
<tr>
<td>9</td>
<td>0.4734</td>
<td>0.0383</td>
<td>0.2253</td>
<td>0.0361</td>
<td>0.2270</td>
<td>0.00038789</td>
<td>0.01232</td>
<td>0.031485</td>
</tr>
<tr>
<td>10</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

* Imaginary numbers, so it is not feasible

Using the estimated values of the mean \( \hat{\mu}_w \) and variance \( \hat{\sigma}^2_w \) portfolio of the Table 6 above can be made efficient surface graphs like Figure 1 below.
Taking into account the results given in Table 6, it appears that any increase in the value of risk tolerance $\tau$, of the 0 to 9 resulted in changes in the composition of the portfolio weights, although very slowly. Also resulted in an increase in the average value of the portfolio is accompanied by an increase in the value of risk, which in this case is measured by the variance. Risk tolerance $\tau > 9$ is not feasible to invest, because the weight will give the composition of the portfolio is imaginary. Viable area to invest in efficient surface indicated by the chart portfolios like Figure 1 above.

4. Conclusion

In this paper has done research on volatility (Islamic stock) that associated causality Composite Stock Price Index (IHSG). The results showed that: There is a one-way causal relationship with stock prices ASII IHSG. IHSG variables affect stock prices ASII. Meanwhile, the stock price variable does not affect the IHSG ASII; there is a one-way causality of the rupiah against the IHSG AALI price. Variable of the rupiah against IHSG AALI effect on stock prices. Meanwhile, the price variable AALI not affect the rupiah against IHSG; there is no causal relationship between stock prices KLBF the rupiah against the IHSG. KLBF stock price variable does not affect the exchange rate against IHSG. Likewise variables rupiah against IHSG does not affect the stock price KLBF; there is no causal relationship between the exchange rate against the share price LSIP IHSG. Variable rupiah against IHSG has no effect on stock prices LSIP. Likewise with LSIP stock price does not affect the exchange rate against IHSG; and there is a two-way causality between the exchange rate against the USD with the stock price PTBA. Variables affect the price of IHSG of PTBA shares. Likewise affect the IHSG Islamic stock price PTBA. Portfolio optimization shows that the greater the risk tolerance of the average yield, the greater the portfolio return, but is followed by the magnitude of the level of risk as measured by the variance. Optimal portfolio is obtained on the risk tolerance of 0.4714, because produces ratio $\hat{\mu}_w / \hat{\sigma}_w^2$ largest. The optimal portfolio weights are obtained when the composition of 0.4714, 0.0391, 0.2257, 0.0375 and 0.2262. That is the average produces of the portfolio returns 0.00038666, the risk (variance) of 0.01202.

References


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Figure 1. An Efficient Frontier Portfolio


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