THE SOLUTION OF FALKNER-SKAN FLOW AND HEAT TRANSFER OVER A WEDGE BY HOMOTOPY ANALYSIS METHOD (HAM)

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Abstract: The study of convective heat transfer has generated many interests and become more important recently because of their wide applications in engineering and in several industrial processes. In this work, the effects of pressure gradient and Prandtl number on velocity and temperature profiles on convective heat transfer in boundary layer over a wedge plate are discussed. The governing boundary layer equations are transformed into a system of non-dimensional equations and then solved using Homotopy Analysis Method (HAM). This method provides the freedom to choose the initial guess function and is used to solve the related boundary layer problem. The approximate analytical results are then compared with published results obtained by Homotopy Perturbation Method (HPM) and Finite Difference Method (FDM). This study is focused on comparing the accuracy of results and applicability of the three methods which are HAM, HPM and FDM. HAM which is an approximate analytical approach is compared with HPM and FDM, and the result reveals that HAM is a powerful tool for the boundary layer problem. Results of HAM in the absence of pressure gradient are better than the HPM and FDM.

Keywords: Laminar Convection, Pressure gradient, Heat transfer, Homotopy Analysis Method (HAM), Homotopy Perturbation Method (HPM), Finite Difference Method (FDM).

1. Introduction

The problem of boundary layer and heat transfer over a flat plate is of great interest to researchers, due to the high applicability of this problem in engineering/industrial disciplines. Such investigations find their applications involving laminar flow heat transfer as in electronic components cooling and plate-type heat exchangers design. Such studies for forced convective in boundary layer include the work of Remie et al., (2007), Guo, et al., (1997), Cossali, (2005), Mukhopadhyay, et al., (2010), Aydin and Kaya (2005), Defraeye et al., (2010), Abd-el-Malek and Helal (2008), Seddeek (2005), and Juncu (2008). More recently, Mahgoub (2013) discussed forced convection heat transfer over a flat plate in a porous medium. Analysis of convective momentum and heat transfer system in boundary layer was done by Escriva and Govannini (2003). The study on heat and mass transfer under various physical situations was carried out by many researchers (e.g. Magyari, et al. (2004), Kandula et al., (2007), Ozgoren et al., (2013), Gireesha et al., (2011), and Yao, et al. (2010)). The study of direct numerical simulation (DNS) of flow over a flat plate was carried out by Wissink and Rodi (2009). Vajravelu et al., (2013) investigated the unsteady convective boundary layer flow of a viscous fluid, and this was an extension work of Aydin and Kaya (2005). The influence of non-Darcian by
Seddeck (2005) was on forced convection heat transfer over a flat plate with temperature dependent viscosity. Abd-el-Malek and Helal (2008) gave the numerical solutions corresponding to the similarity solutions for magneto-force-unsteady free convection laminar boundary-layer flow. Forced convection heat transfer leading from the flow of a uniform stream over a flat surface was considered by Merkin and Pop (2011).

In view of this, the study of force-convective temperature wall function over a flat plate was carried out by many researchers (Defraeye et al., (2010), Mahgoub (2013), Karava et al. (2011), Bhattacharrrya et al. (2011), Mirgolbabaei et al., (2010), Tao et al., (2002), and Nadeem et al. (2013)). Application of He’s homotopy perturbation method (HPM) to solve the problem of heat transfer and convection equations was carried out by many researchers (Ganji and Sadighi (2006), Rajabi et al. (2007), Fathizadeh and Rasidi (2009)). Dalgliesh and Surry (2003) applied HAM to the boundary layer wind tunnel. The problem of free convection about a vertical flat plate was solved by Xu (2004) using HAM. Hayat and Sajid (2007) applied HAM on MHD boundary layer flow of an upper-convected Maxwell fluid. Heat transfer analysis of unsteady boundary layer flow was done by Mehmood et al. (2008), also via HAM. Many other researchers also employed HAM in the study of boundary layer problem (e.g. Kumari and Nath (2009), Niu and Wang (2009), Sajid et al. (2009), Turkyilmazoglu (2009), Zabakhsh et al. (2009), Domairry and Nadim (2008), Chowdhury et al. (2009), Odibat (2010), and Hayat et al. (2012)).

Pressure gradient gives a great importance on boundary layer convective heat transfer process (Miyake et al. (1995)). Umur (2000) investigated flow and heat transfer features on laminar flows with pressure gradients. Rebay and Pedek (2005) worked on effects of pressure gradient on unsteady force convection. Boundary layer convective heat transfer with pressure gradient was studied by Fathizadeh and Rashidi (2009) using HPM. The numerical study of temperature distribution in the luminary boundary layer subjected to pressure gradient is done by Afzal and Maqbool (2010).

The solutions to the Falkner-Skan equation are known as wedge flow solutions. Abbasbandy and Hayat (2009) work on solution of MHD Falkner-Skan flow using HAM. Solution of Falkner-Skan equation for wedge was reported by Alizabeh et al. (2009), Yao (2009) and Kuo (2003). An extension was made by Pantokratoras (2006) on the work of Kuo (2003) to constant wall temperature and variable viscosity on Falkner-Skan velocity. A numerical approach to the solution of Falkner-Skan equation was presented by Asaithambi (1997, 1998), and also Brodie and Banks (1986) studied on further properties on Falkner-Skan equation.

The present problem (without heat transfer aspect) was first considered by Falkner and Skan in 1931, sometimes referred to as *wedge flow*. When \( \eta = 0 \), this problem reduces to the Blasius flow over a flat plate. The case \( \eta = 1 \) is for the wedge half-angle \( 90^\circ \), which is the two-dimensional
stagnation flow known as Hiemenz flow. There have been many theoretical models developed to describe laminar forced convection over a flat plate. However, to the best of our knowledge, no investigation has been made yet to analyse the effects of pressure gradient on steady forced convection using HAM. The non-linear systems are solved using HAM proposed by Liao (2003).

2. Mathematical Formulation

A steady laminar boundary-layer problem is studied. Let the free stream velocity $U_\infty$ which is not constant and depends on the pressure gradient along the flat plate. Assume the free stream temperature $T_\infty$ be constant and let all fluid properties be constant. The velocity distribution is unchanged by the temporal changes in temperature. The boundary layer flow over a flat plate is governed by the equations: continuity and Navier-Stokes equations. The associated partial differential equations are:

\[
\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0
\]  

\[
u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{dP^*}{dx^*} + \frac{\partial^2 u^*}{\partial y^*^2}
\]

\[
u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = k \frac{\partial^2 T^*}{\partial y^*^2}
\]

subject to the boundary conditions:

\[u^*(0,y^*) = 1, u^*(x^*,0) = 0, \quad u^*(x^*,\infty) = 1, v^*(x^*,0) = 0
\]

\[T^*(0,y^*) = 1, T^*(x^*,0) = 1, T^*(x^*,\infty) = 1
\]

where $x^*$ and $y^*$ are the Cartesian coordinates measured along the surface of the flat plate starting from the leading edge of the flat plate. The $y^*$ coordinate measured normal to the flat plate $u^*$ and $v^*$ are the velocity components along $x^*$ and $y^*$ directions, while $T$ is the fluid temperature.

We introduce the following dimensionless variables as follows:

\[u = \frac{u^*}{U_\infty}, \quad v = \frac{v^*}{U_\infty}, \quad P = \frac{P^*_\rho}{\rho U_\infty^2}, \quad \theta = \frac{T - T_\infty}{T_\tau - T_\infty}, \quad x = \frac{x^*}{L}, \quad y = \frac{y^*}{L}
\]

The dimensionless equations are obtained as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]
We introduce the stream function \( \psi(x, y) \) is defined as:

\[
\begin{align*}
    u &= \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}.
\end{align*}
\]

Converting the set of partial differential equations (6)–(9) into ordinary differential equations by using the similarity transformation

\[
\eta = y \sqrt{\frac{U_\infty}{v x}} , \quad \psi = \sqrt{v x U_\infty} f(\eta) ,
\]

where \( \eta \) is the similarity variable, \( f \) is the similarity function and \( \psi \) is the stream function, and simply by replacing \( u \) and \( v \) components of velocity by a single function. Let define the free stream velocity as \( U_\infty \sim x^m \) where \( m \) is the Falkner-Skan power-law parameter. The quantity \( \beta \) in (11) is related to the wedge angle, where the wedge angle is given by \( \beta \pi / 2 \). The case \( m = 0 \) is for a flat plate, and \( m = 1 \) is for the wedge half-angle \( 90^0 \), which is two-dimensional stagnation flow known as Hiemenz flow;

\[
m = \frac{\beta}{2-\beta} \quad \text{and} \quad \beta = \frac{2m}{m+1} .
\]

By substitution, we use equations (10) & (11) in equation (6)-(9), and thus these become;

\[
f''' + \frac{m+1}{2} f'' + m(1-f'^2) = 0 \quad \text{(12)}
\]

\[
\theta'' + \frac{\text{Pr} (m+1)}{2} f \theta' = 0 \quad ,
\]

with the boundary conditions

\[
\begin{align*}
    f(0) &= 0, \quad f'(0) = 0, \quad f'(\eta_\infty) = 1 \\
    \theta(0) &= 1, \quad \theta(\eta_\infty) = 0,
\end{align*}
\]

(14)
Where $f' = \frac{u}{U_\infty}$ and $\eta_\infty$ is a function of pressure gradient and Prandtl number respectively.

3. The Homotopy Analysis Method (HAM)

In this section, HAM is applied to solve equations (12) & (13) subjected to boundary conditions (14). We assume that $f(\eta)$ and $\theta(\eta)$ can be expressed by a set of functions

\[ \left\{ c_{mn} \eta^{2m} \mid m \geq 1, n \geq 0 \right\} (15) \]

in the forms

\[ f(\eta) = \sum_{m=1}^{+\infty} \sum_{n=0}^{+\infty} a_{mn} \eta^{2m}, \quad (16) \]

\[ \theta(\eta) = b_{0,0} + \sum_{m=1}^{+\infty} \sum_{n=0}^{+\infty} b_{mn} \eta^{2m}, \quad (17) \]

where $a_{mn}$ and $b_{mn}$ are coefficients, and from the boundary conditions (equation (14)) we choose

\[ f_0(\eta) = \eta + e^{-\eta} - 1, \quad (18) \]

\[ \theta_0(\eta) = e^{-\eta}, \quad (19) \]

as the initial approximations of $f(\eta)$ and $\theta(\eta)$. In equations (20) & (21) $L$ is the auxiliary linear operator defined by

\[ L_1[f] = f''' - f' \quad , \quad (20) \]

\[ L_2[f] = \theta'' - \theta \quad , \quad (21) \]

the auxiliary linear operators satisfying

\[ L_1 [c_1 \eta^2 + c_2 \eta + c_3] = 0 \quad , \quad (22) \]

\[ L_2 [c_4 \eta + c_5] = 0 \quad . \quad (23) \]

Here $c_i$ ($i = 1, 2, 3, 4, 5$) are constants. Based on equations (12) and (13), the non-linear operators

\[ N_i[f(\eta); q] = \frac{\partial f(\eta; q)}{\partial \eta^2} + m \left( 1 - \frac{\partial f(\eta; q)}{\partial \eta} \right)^2 \]

\[ \frac{1}{2} f(\eta; q) \frac{\partial^2 f(\eta; q)}{\partial \eta^2} + m \left( 1 - \frac{\partial f(\eta; q)}{\partial \eta} \right)^2 \]
where \( q \in [0, 1] \) is an embedding parameter, and \( \tilde{f}(\eta; q) \) and \( \tilde{\theta}(\eta; q) \) are mapping functions for \( f(\eta) \) and \( \theta(\eta) \) respectively. From the operators, we can construct the zeroth-order deformation equations as

\[
(1 - q)\mathcal{L}_1[\tilde{f}(\eta; q) - f_0(x)] = qhN_1[\tilde{f}(x; q)] , \quad (26)
\]

\[
(1 - q)\mathcal{L}_2[\tilde{\theta}(\eta; q) - \theta_0(x)] = qhN_2[\tilde{f}(x; q), \tilde{\theta}(\eta; q)] . \quad (27)
\]

Here we consider the auxiliary function as

\[
H(x) = 1 , \quad (28)
\]

where \( h \) is an auxiliary non-zero parameter. The boundary conditions for equations (14) are presented as

\[
\hat{f}(0; q) = 0, \quad \hat{f}'(\eta; q) = 0, \quad \hat{f}'(\eta_\infty; q) = 1 , \quad (29)
\]

\[
\hat{\theta}(0; q) = 1, \quad \hat{\theta}(\eta_\infty; q) = 0 . \quad (30)
\]

Clearly, when \( q = 0 \) and \( 1 \), the above zeroth-order deformation equations have the following solutions,

\[
\hat{f}(\eta; 0) = f_0(\eta), \quad \hat{\theta}(\eta; 0) = \theta_0(\eta) , \quad (31)
\]

\[
\hat{f}(\eta; 1) = f(\eta), \quad \hat{\theta}(\eta; 1) = \theta(\eta) . \quad (32)
\]

When \( q \) increases from 0 to 1, \( \hat{f}(\eta; q) \) and \( \hat{\theta}(\eta; q) \) vary from \( f_0(\eta) \) and \( \theta_0(\eta) \) to \( f(\eta) \) and \( \theta(\eta) \) respectively. From Taylor’s theorem and Equations (31) and (32), we obtained

\[
\hat{f}(\eta; 0) = f_0(\eta) + \sum_{m=1}^{+\infty} f_m(\eta) q^m , \quad (33)
\]

\[
\hat{\theta}(\eta; 0) = \theta_0(\eta) + \sum_{m=1}^{+\infty} \theta_m(\eta) q^m , \quad (34)
\]

where
The convergence of the series solutions (33) and (34) depend upon the choice of auxiliary parameter \( h \). Assume that \( h \) is chosen such that the series solutions (33) and (34) are convergent at \( q = 1 \), then due to equations (31) and (32) we obtain

\[
f(\eta) = f_c(\eta) + \sum_{m=1}^{+\infty} f_m(\eta),
\]

\[
\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{+\infty} \theta_m(\eta). \tag{38}
\]

For the \( m \)th-order deformation equations, equations (26) and (27) are differentiated up to \( m \) times with respect to \( q \) and divide by \( m! \) and then set \( q = 0 \). The resulting deformation equations at the \( m \)th-order are

\[
\mathcal{L}_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h R_{1,m}(\eta), \tag{39}
\]

\[
\mathcal{L}_2[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = h R_{2,m}(\eta), \tag{40}
\]

with the following boundary conditions

\[
f_m(0) = 0, \quad f_m'(0) = 0, \quad f_m'(\eta_\infty) = 0, \tag{41}
\]

\[
\theta_m(0) = 0, \quad \theta_m(\eta_\infty) = 0, \tag{42}
\]

where

\[
R_{1,m}(\eta) = f''''_{m-1}(\eta) + \frac{m + 1}{2} \sum_{k=0}^{m-1} f'_{m-1-k} f''_k - m \sum_{k=0}^{m-1} \sum_{i=0}^{k} (f'_{m-1-k} f''_{k-i-1}), \tag{43}
\]

\[
R_{2,m}(\eta) = \theta''''_{m-1}(\eta) + \frac{Pr (m + 1)}{2} f_{m-1} \theta''_{m-1}, \tag{44}
\]

and

\[
\chi_m = \begin{cases} 
0, & m \leq 1 \\
1, & m > 1
\end{cases} \tag{45}
\]
The solutions of equations (43) and (44) can be obtained easily by symbolic computation software such as Mathematica, Maple, Matlab etc.

4. Convergence of the HAM solution

In this section we will discuss the convergence of our solutions. The analytic solution contains the auxiliary parameter \( h \), which influences the convergence region and rate of approximation for the HAM solution. We have seen that, if the series are convergent, then the series represent converged solutions to the exact solutions. Liao (2003), examines the behaviour of the exact solutions as a function of the parameter \( h \), once the homotopy series approximating the exact solution is computed. This is called the \( h \)-curves approach. The constant \( h \)-curve is quite rational, whenever convergence takes place at \( p = 1 \), the quantities of exact solution should be free of the parameter \( h \). The analytical expressions (26) – (27) and (39) – (40) contain the auxiliary parameter \( h \). As pointed out by Liao (2007), that the convergence region and rate of approximations given by HAM are strongly dependent upon \( h \). Figs. 1 & 2 show the \( h \)-curves to find the range of \( \hat{h} \) for velocity profile is \(-0.45 \leq \hat{h} \leq 0.45\) and temperature profile is \(-1.5 \leq \hat{h} \leq 1.5\). We take \( Pr = m = 1 \). It is apparent from fig.1 that the range for admissible values for \( h \) is \(-0.1 \leq \hat{h} \leq 0.1\) for the velocity profile and \(-0.4 \leq \hat{h} \leq 0.4\) for the temperature profile.

![Figure 1: \( h \)-curve for the velocity profile for boundary layer at 6th order approximation](image-url)
Figure 2: $h$ - curve for the temperature profile for boundary layer at 6th order approximation

Table 1: Difference between 6th order HAM and FDM with HPM solutions $f(\eta)$

| $\eta$ | $|\text{HAM} - \text{FDM}|$ | $|\text{HPM} - \text{FDM}|$ |
|-------|----------------|----------------|
| 0     | 0              | 0              |
| 0.2   | 0.0000021      | 0.0011520      |
| 0.4   | 0.0000018      | 0.0027098      |
| 0.6   | 0.0000061      | 0.0050352      |
| 0.8   | 0.0000128      | 0.0077411      |
| 1     | 0.0000276      | 0.0109847      |
| 1.2   | 0.0000733      | 0.0147542      |
| 1.4   | 0.0000201      | 0.0122162      |
| 1.6   | 0.0000303      | 0.0238670      |
| 1.8   | 0.0000510      | 0.0291937      |
| 2     | 0.0000287      | 0.0350039      |
| 2.2   | 0.0000227      | 0.0412504      |
| 2.4   | 0.0000019      | 0.0478580      |
| 2.6   | 0.0000059      | 0.0547154      |
| 2.8   | 0.0000073      | 0.0616990      |
| 3     | 0.0000218      | 0.0685256      |
| 3.2   | 0.0000249      | 0.0750468      |
| 3.4   | 0.0000259      | 0.0809684      |
| 3.6   | 0.0000449      | 0.0860158      |
| 3.8   | 0.0000302      | 0.0899315      |
| 4     | 0.00000336     | 0.0925112      |
| 4.2   | 0.0000304      | 0.0936436      |
| 4.4   | 0.0000205      | 0.0933513      |
| 4.6   | 0.00000510     | 0.0975944      |
| 4.8   | 0.0000740      | 0.0894515      |
| 5     | 0.0000135      | 0.0867954      |
Table 2: Difference between 6th order HAM and FDM with HPM solutions $f''(\eta)$

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Table 3: Difference between 4th order HAM and FDM with HPM solutions $f''(\eta)$

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5. Results and Discussion

The absolute errors between the 6-th order HPM solutions reported by Fathizadeh and Rashidi (2009) and the 6-th order HAM and the FDM solutions in the absence of pressure gradient (η = 0) are presented. It is observed that the HAM solutions are accurate enough as compared with the HPM solutions. The results are illustrated in Tables 1, 2, 3 and 4. In Cebeci (1988), we saw the effect of pressure gradient (m) reported by Blasius 1908 for a range of (m) as m = -0.0904 to 0. The case m = 0 is for the flat plate (Blasius flow). In our present work, it is possible to obtain the separation point by obtaining the velocity equation. From equations (12) and (13), when \( f''_w \) become zero; the value of the shear at the wall will become zero too. Table 4 shows comparison solution of the Falkner-Skan equation for zero mass transfer of the values of \( f'' \) obtained for the pressure gradient range. In this table 4, the separation point using HAM method happens at \( \eta = -0.091 \) which is in good agreement with the work reported by Blasius 1908 see Cebeci (1988).

The first finding we have is on the velocity profile with different pressure gradient values. Solutions for a range of pressure gradient parameter (m) are, however, useful since these illustrate the main characteristics of the response of boundary layers to pressure gradients. The pressure gradient values that are used in the present study are \(-0.091, -0.065, 0, 0.3333, \) and \( 1 \). The pressure gradient with value \( 1 \) (wedge half-angle 90°) in the present work represents two-dimensional stagnation flow. And also, the pressure gradient with value \(-0.091\) represents the separation point. It is found that for \( f''_w \equiv 0 \), the wall shear stress \( \tau_w = 0 \) for \( m = -0.091 \) and the boundary layer is on the point of separation on at all x. In the heat transfer problem the pressure gradient effect in the flow is different from that in a boundary-layer flow. Large pressure gradient hinders boundary layer growth and resulted in a reduced boundary layer thickness. Figure 3 illustrates the influence of the different pressure gradient parameters on \( f''(\eta) \) for forced convection flow. In Figure 3, when the pressure
gradient parameter is $m = -0.091$, the separation point is reached and the fluid will not be in contact with the surface anymore. It is also observed that the velocity increases with the increase in the pressure gradient parameter $m$. The pressure gradient parameter suppresses the boundary layer growth, and when the pressure gradient parameter is $m = 1$, this is on a two-dimensional stagnation flow. The boundary layer thickness is reduced due to the presence of pressure gradient on the velocity profile. Based on our results, we can conclude that for $m = -0.091$ the separation point is reached on the velocity profile; see Fig.3. This result is similarly obtained by Fathizadeh and Rasidi (2009).

The influence of Prandtl number parameter ($Pr$) and pressure gradient ($m$) on $\theta(\eta)$ (i.e. temperature profile) is investigated for forced convection flow. The second finding is on the temperature profile with different Prandtl number ($Pr$) with values: $0.6$, $1$, and $15$. The Prandtl number with value 15 represents a large Prandtl number which means that heat wave penetration is less in fluid for example in oil. The Prandtl number with value 0.6 for example liquid metals (Mercury), i.e. a small Prandtl number shows that the fluid takes some distortions in its temperature profile. In convective heat transfer, the Prandtl number controls the relative thickness of the momentum and the thermal boundary layer. For a favourable Prandtl number, the heat is transferred slower, which makes the temperature drops slowly. For a small Prandtl number, the temperature shows a sharp fall. The thermal boundary layer thickness decreases sharply with significant increases in Prandtl number $Pr$. The reason is that, the value of Prandtl number is larger, the thermal diffusivity decreases. It will result in decrease of energy transfer ability and causing the thermal boundary layer to decrease. In Figures 4, 5 and 6 respectively, show the effect of the different Prandtl parameters $Pr$ for fixed pressure gradient $m$ on the temperature profiles. Obviously, from Figure 4, we can see that the temperature for fixed $m = 0$ (absence of pressure gradient). It is shown that decreasing the Prandtl number increases the thermal boundary layer thickness that is in the thermal boundary layer, temperature at every location reduces. For a larger Prandtl number, it has a relatively lower thermal diffusivity. Therefore this greatly increases the temperature and the thermal boundary layer thickness. Figures 5 and 6, show the influence of pressure gradient for different values of Prandtl parameter. In the presence of pressure gradient effect, which is shown in Figures: 5, 6 and 7 on the flow, is quite different from that in a boundary-layer flow Figure 4. In Figure 7 when pressure gradient parameter $m = 1$, it suppresses the boundary layer growth which leads to reduction of boundary layer thickness. The effect is more on higher values of pressure gradient parameter $m$.

The influence of the different pressure gradients $m$ with various values of Prandtl parameter $Pr$, on temperature profile is shown in Figures 8-10. We observed that temperature increases with the decrease in the values of pressure gradient parameter $m$. The boundary layer thickness decreases by
increasing the pressure gradient parameter $m$. This behaviour shows that, when the pressure gradient $m = 1$ the main change of temperature in fluid is confined in region close to surface. For low value of pressure gradient $m = -0.065$, the heat transfer from the plate to fluid happens on large distance into the fluid. Now for fixed values of $Pr = 0.6$, $Pr = 1$, and $Pr = 15$ with varying pressure gradients parameter $m$ that is $(m = -0.065, \ 0, \ 0.3333, \ & 1)$. Increasing the pressure gradient values results in decreasing of boundary layer thickness and increasing in temperature distribution in the fluid. We observed that from the Figures 8-10 that by increasing Prandtl parameter $(Pr)$, the thermal boundary layer thickness $(\delta_t)$ decreases. See Figure 10 for a given value of Prandtl number $Pr = 15$, the gradient of curves are steeper which means the fluid temperature approaches the wall temperature at initial stages. This resulted to thermal boundary layer thickness becomes thinner. The temperature profiles reveal that for low value of Prandtl number the boundary layer thickness decreases as pressure gradient increases. The effect of pressure gradient variation on temperature profile for a fixed Prandtl number $Pr$ on the thermal boundary layer thickness decreases as the Prandtl number $Pr$ increases. At low Prandtl number $Pr = 0.6$ in Figure 8 and higher Prandtl number $Pr = 15$ in Figure 10, for fixed Prandtl number $Pr$ by increasing the pressure gradient the thermal boundary thickness decreases significantly. We observed that increasing pressure gradient increases the tendency of fluid temperature to reach the surface temperature.

In Figures 3-10, it is observed that increasing of pressure gradients results in decreasing the surfaces temperature, and therefore in the boundary layer thickness. For fixed value of $m$ the ratio of hydrodynamic boundary layer thickness to thermal boundary layer thickness $(\delta/\delta_t)$ increases by increasing Prandtl number $Pr$. 

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Figure 3: Velocity Profile $f'$ for various values of $m$

Figure 4: Temperature Profile $\theta$ for various values of $Pr$ at $m = 0$
Figure 5: Temperature Profile $\theta$ for various values of $Pr$ at $\gamma_i = -0.065$

Figure 6: Temperature Profile $\theta$ for various values of $Pr$ at $\gamma_i = \frac{5}{3}$
Figure 7: Temperature Profile $\theta$ for various values of $Pr$ at $m = 1$

Figure 8: Temperature Profile $\theta$ for various values of $\gamma$ at $Pr = 0.6$
Figure 9: Temperature Profile $\theta$ for various values of $\eta$ at $Pr = 1$

Figure 10: Temperature Profile $\theta$ for various values of $\eta$ at $Pr = 15$
6. Conclusion

In this research, HAM is applied to the solutions of the boundary layer force convective heat transfer. Comparisons between the standard HPM solution and HAM solution with the FDM solutions are made. For the six-order HAM solutions, it is found to achieve good accuracy as compared to HPM solutions.

Based on the results obtained, these show how pressure gradient \( m \) and Prandtl number \( Pr \) affect the flow and heat transfer features in non-Newtonian fluid. From the present analytical approach, the important findings from the graphical analysis of the results are:

1. Increasing the pressure gradient parameter \( m \) resulted in significantly decrease of momentum boundary layer thickness.
2. An increasing pressure gradient increases the tendency of fluid temperature to approach the surface temperature.
3. When the Prandtl number \( Pr \) is high, the gradient of curves are steeper which signifies that the fluid temperature gets to the wall temperature at initial stage and the thermal boundary layer thickness become thinner.
4. Due to increasing convective force parameter, the velocity increases and the temperature decreases.
5. An increase of Prandtl number reduces the velocity along the plate as well as the temperature.
6. The hydrodynamic \( \eta_{nf} \) varies with different pressure gradients and Prandtl number.
7. The effect of pressure gradient in the number \( Nu \) (heat transfer coefficient) is in reducing the higher Prandtl number \( Pr \).

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References


