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# The APT with Lagged, Value-at-Risk and Asset Allocations by Using Econometric Approach

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**Abstract :** In this paper analyzed the *Arbitrage Pricing Theory* with lagged, Value-at-Risk and asset allocations by using economic approach. It is assumed that stocks were analyzed following the model of *Arbitrage Pricing Theory* with lagged. Where the factor risk premium in the past affects the present changes in stock return. The return of factor is assumed to have non constant volatility and there is effect of long memory. Long memory effects are estimated using the rescaled range method (R/S) or Geweke and Porter-Hudak (GPH) method. The mean and non constant volatility is estimated using ARFIMA-GARCH models. The portfolio risk level is measured by the Value-at-Risk (VaR). Asset allocation problem solving carried out using the Lagrangian multiplier technique and the Kuhn-Tucker method. The purpose of this research forms the efficient portfolio and determines optimal portfolio weights. Empirical research conducted on some stocks that are traded in capital markets in Indonesia.

**Keywords:** APT, ARFIMA, GARCH, Value-at-Risk, Asset Allocation.

**Abstrak.** Dalam paper ini dianalisis *Arbitrage Pricing Theory* dengan *lagged*, *Value-at-Risk*, dan alokasi aset dengan menggunakan pedekatan ekonometri. Diasumsikan saham-saham yang dianalisis mengikuti model *Arbitrage Pricing Theory* dengan *lagged*. Di mana premi risiko faktor pada masa lalu berpengaruh terhadap perubahan *return* saham masa sekarang. *Return* faktor diasumsikan memiliki volatilitas tak konstan dan terdapat efek *long memory*. Efek *long memory* diestimasi menggunakan metode *rescale range* (R/S) atau Geweke dan Porter-Hudak (GPH). Rata-rata dan volatilitas tak konstan diestimasi menggunakan model-model ARFIMA-GARCH. Tingkat risiko portofolio diukur berdasarkan *Value-at-Risk*. Penyelesaian masalah alokasi aset dilakukan menggunakan teknik Lagrangian multiplier dan metode Kuhn-Tucker. Tujuan penelitian ini membentuk portofolio efisien dan menentukan bobot portofolio optimum. Penelitian secara empiris dilakukan pada beberapa saham yang diperdagangkan dalam pasar modal di Indonesia.

**Kata Kunci:** APT, ARFIMA, GARCH, *Value-at-Risk*, Alokasi Aset.

## 1. Introduction

Formulation of *Arbitrage Pricing Theory* (APT) has important implications in determining stock prices [11]. It's stated that the return of a stock (or portfolio) will be affected by one or several explanatory variables (factor index). However, APT does not mention (explicitly) what variables affect the stock return. To determine the factors that influence the degree of sensitivity and magnitude of asset returns to each factor, is to set a number of factors that allegedly had an influence on stock return [7; 11]. These factors include the industrial variables (egg market index, alternative products, etc.) and economic variables (egg inflation, interest rates, etc.) [10; 11].

APT applies the law of one price, in equilibrium, the relationship between risk and stock return occurs in one area (if there is more than one factor). This situation can be achieved through a process of arbitrage. Arbitrators will cause all the portfolio is located in one and the same area [7; 11]. The location of each portfolio will be determined by the proportion (weight) of funds invested in the establishment of a portfolio [7; 8]. Determination of the proportion (weight) portfolio is a problem that should be sought the solution. To determine the proportion (weight) can be conducted using a


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portfolio optimization [9; 12; 15; 17]. The composition obtained proportions will affect the return expectations and risk portfolio [2; 7; 15]. Highly popular portfolio risk is measured using a Value-at-Risk (VaR) [1; 3; 4; 6; 20].

In this paper, the formulation of APT as a means of determining the stock price will be expanded by considering the factors of lagged. Where the factors in that last period is assumed to influence the formulation of APT. The return of factors in APT is assumed to have non constant volatility and there is an effect of long memory. Non constant volatility and long memory effect will be analyzed using ARFIMA-GARCH models [14; 16; 18; 21; 22]. The mean and variance of returns of stocks estimated based on the APT with the lagged, which has non constant volatility and long memory effects. Using the mean estimator and the variance will be arranged the problem of Asset allocation. Asset allocation is based on the mean-VaR approach [5; 8; 13; 19]. Settlement asset allocation problem based on Lagrangian multiplier techniques and methods of the Kuhn-Tucker [5; 7]. Thus the analysis could be performed, because many stocks have characteristics such as the discussion here. The aim is to establish an efficient portfolio and determine the proportion (weight) portfolio optimally. The empirical research carried out on a few stocks that are traded at the capital markets in Indonesia.

## 2. Methodologies

***Determining stock return.*** Let $P_{it}$ and $r_{it}$ denote the prices and the returns of stock $i$ ($i=1,...,N$ and $N$ is the number of stocks that are analyzed), respectively, at the time $t$ ($t=1,...,T$, $T$ denotes the period of data observation). Stocks return $r_{it}$ is calculated using the formula $r_{it}=\ln(P_{it}/P_{it-1})$. Let $F_{jt}$ and $\hat{r}_{jt}$ respectively denote the price and the return of factor index $j$ ($j=1,...,M$ and $M$ is the number of factors index in the APT), at the time $t$, $t=1,...,T$. In the same way to calculate $r_{it}$, the factors index return $\hat{r}_{jt}$ are calculated by $\hat{r}_{jt}=\ln(F_{jt}/F_{jt-1})$ [6; 21].

### 2.1 Mean Modeling

In the next stage we identify the existence of long memory effect in the data return of factor index using the rescale range method (R/S) or Geweke and Porter-Hudak (GPH) method. The parameter estimation of fractional difference index $d_j$, $j=1,...,M$, is performed using the maximum likelihood method [14; 16; 21; 22]. The confidence interval $(1-c)100\%$ for $d_j$ is $\hat{d}_j - z_{c/2}.\sigma_{d_j} < d_j < \hat{d}_j + z_{c/2}.\sigma_{d_j}$ where $\hat{d}_j$ denotes estimator of $d_j$, and $z_c$ denotes the percentile of standard normal distribution at the significance level $c$. Let $\mu_{d_j}$ and $\sigma_{d_j}$ respectively denote the mean and standard deviation of $d_j$. We can test the null hypothesis $H_0:\hat{d}_j=0$ against $H_1:\hat{d}_j\neq 0$ using statistic $z_{d_j}=(d_j-\mu_{d_j})/\sigma_{d_j}$. We reject $H_0$ if the value $z_{d_j}<z_{c/2}$ or $z_{d_j}>z_{1-c/2}$ [16; 21].

Fractional difference process is defined as:

$$(1-B)^{d_j}\hat{r}_{jt}=a_{jt}, -0.5<d_j<0.5; \tag{1}$$

where $\{a_{jt}\}$ is the error component which is the white noise process, and $B$ denotes the backshift operator? If the sequence of fractional difference $(1-B)^{d_j}\hat{r}_{jt}$ is following the model of ARMA($p,q$), then we call $\hat{r}_{jt}$ *autoregressive fractionally integrated moving average degree* $p$, $d$ and $q$ process, or ARFIMA($p,d,q$) [16; 21; 22]. The ARMA($p,q$) follows the following form

$$\hat{r}_{jt} = \psi_{j0} + \sum_{g=1}^{p} \psi_{jg} \hat{r}_{jt-g} + a_{jt} + \sum_{h=1}^{q} \theta_{jh} a_{jt-h} , \tag{2}$$

with $\psi_{j0}$ constant and $\psi_{jg}$ $(g = 1,..., p)$ and $\theta_{jh}$ $(h = 1,..., q)$ the parameter coefficients of mean model of factors index return $j$, $j = 1,..., M$ . We assume that $\{a_{jt}\}$ is the error sequence of white noise process with mean zero and variance $\sigma^2_{a_j}$ [20; 21; 22].

Stages of mean modeling process include: (i) Identification of the model, (ii) parameters estimation, (iii) diagnostic tests, and (iv) Prediction [21].

## 2.2 Non Constant Volatility Modeling

The non constant volatility of the returns of factor index is modeled using *generalized autoregressive conditional heteroscedastic* (GARCH) models. Suppose $\mu_{jt}$ and $\sigma^2_{jt}$ respectively denote the mean and non constant volatility of return of factor index $j$ ( $j = 1,..., M$ and $M$ denotes the number of factors index in the APT), at the time $t$ ( $t = 1,..., T$ and $T$ is the period of data observation). The error $a_{jt}$ can be calculated as $a_{jt} = r_{jt} - \mu_{jt}$ [21; 22]. The non constant volatility $\sigma^2_{jt}$ will follow the GARCH model of degree $m$ and $n$ or GARCH( $m,n$ ), if

$$a_{jt} = \sigma_{jt} \varepsilon_{jt} , \sigma^2_{jt} = \alpha_{j0} + \sum_{k=1}^{m} \alpha_{jk} a^2_{jt-k} + \sum_{l=1}^{n} \beta_{jl} \sigma^2_{jt-l} + \varepsilon_{jt} . \tag{3}$$

where $\alpha_{j0}$ is a constant and $\alpha_{jk}$ $(k = 1,..., m)$ and $\beta_{jl}$ $(l = 1,..., n)$ denote the parameter coefficients of non constant volatility model of factor index return $j$ ( $j = 1,..., M$ ). Here we assume $\{\varepsilon_{jt}\}$ is the sequence independent and identically distribution (iid) random variable with mean zero and variance 1, $\alpha_{j0} > 0$ , $\alpha_{jk} \geq 0$ , $\beta_{jl} \geq 0$ , and $\sum_{k=1}^{\max(m,n)} (\alpha_{jk} + \beta_{jk}) < 1$ [21; 22].

The stages of non constant volatility modeling include: (i) The estimation of mean model, (ii) Testing the effect of ARCH, (iii) Model identification, (iv) Non constant volatility model estimation, (v) Diagnostic test, and (vi) Prediction [21].

We further use the mean model (2) and the non constant volatility model (3), to calculate $\hat{\mu}_{jt} = \hat{r}_{jT}(1)$ $\hat{\sigma}^2_{jt} = \hat{\sigma}^2_{jT}(1)$ , i.e. the 1-step ahead prediction after time period $T$ of the mean and the variance [21].

## 2.3 Modeling of Stock Return under APT with Lagged

In this section expand the APT to APT model with lagged. It is known that $r_{it}$ the return of stock $i$ at the time $t$ , and $\hat{r}_{jt}$ returns the index factor $j$ at the time $t$ . Suppose $\tilde{r}_t$ is the risk free asset return at the time $t$ ( $t = 1,..., T$ and $T$ the period of data observation). APT regression model with lagged expressed as equation

$$\begin{aligned} r_{it} - \tilde{r}_t = \omega_{i0} &+ \omega_{i10}(\hat{r}_{1t} - \tilde{r}_t) + \omega_{i11}(\hat{r}_{1t-1} - \tilde{r}_{t-1}) + \omega_{i12}(\hat{r}_{2t-2} - \tilde{r}_{t-2}) + ... + \omega_{i1L}(\hat{r}_{1t-L} - \tilde{r}_{t-L}) + ... \\ &+ \omega_{i20}(\hat{r}_{2t} - \tilde{r}_t) + \omega_{i21}(\hat{r}_{2t-1} - \tilde{r}_{t-1}) + \omega_{i22}(\hat{r}_{2t-2} - \tilde{r}_{t-2}) + ... + \omega_{i2L}(\hat{r}_{2t-L} - \tilde{r}_{t-L}) + ... \\ &+ \omega_{iM0}(\hat{r}_{Mt} - \tilde{r}_t) + \omega_{iM1}(\hat{r}_{2t-1} - \tilde{r}_{t-1}) + \omega_{iM2}(\hat{r}_{2t-2} - \tilde{r}_{t-2}) + ... + \omega_{iML}(\hat{r}_{Mt-L} - \tilde{r}_{t-L}) + u_{it} , \end{aligned}$$

or it can be written into

$$r_{it} - \tilde{r}_t = \omega_{i0} + \sum_{j=1}^{M} \sum_{\zeta=0}^{L} \omega_{ijl}(\hat{r}_{jt-\zeta} - \tilde{r}_{t-\zeta}) + u_{it} . \tag{4}$$

Assumed that $\{u_{it}\}$ is the white noise of regressions residual [7; 11]. Where $\omega_{i0}$ and $\omega_{ij\zeta}$ ( $i = 1,...,N$ ; $j = 1,...,M$ ; $\zeta = 0,...,L$ and $L$ is length of lagged), respectively declare constants and parameter coefficients of regression for the APT with lagged of stock return $i$ at the time $t$. To estimate the constant $\omega_{i0}$ and parameter coefficients $\omega_{ijl}$ regression of equation (4) can be performed using the least squares method. Length of lagged estimated based on the Ad-Hoc method, namely by looking at the consistency changes of parameter coefficients sign, positive (+) continue or the negative (-) continued, when lagged extended. Referring Blume in 1971, the ability of the index factors explain changes in individual stock returns ranging between 25%-51% rate coefficient of determination [7].

As previously described, $\hat{\mu}_{jt}$ and $\hat{\sigma}^2_{jt}$ successively states the mean and variance of the index return factor $j$ at the time $t$. Suppose that $\tilde{\mu}_t$ and $\tilde{\sigma}^2_t$, in succession states mean and variance of return risk-free asset. Based on the equation (4), the mean stock return $i$ at the time $t$, which $\mu_{it}$ can be estimated using the following equation:

$$\mu_{it} = E(r_{it}) = \tilde{\mu}_t + \omega_{i0} + \sum_{j=1}^{M}\sum_{\zeta=0}^{L} \omega_{ij\zeta}(\hat{\mu}_{jt-\zeta} - \tilde{\mu}_{t-\zeta}) . \tag{5}$$

It is assumed that $E[(\hat{r}_{jt-\zeta} - \tilde{r}_{t-\zeta}),(\hat{r}_{j't-\zeta'} - \tilde{r}_{t-\zeta'})] = 0$ , where $j, j' = 1,...,M$ , $j \neq j'$ and $\zeta, \zeta' = 0,...,L$ , $\zeta \neq \zeta'$. Stock return variance $i$ at the time $t$, that $\sigma^2_{it}$ can be estimated using the equation

$$\sigma^2_{it} = Var(r_{it}) = \tilde{\sigma}^2_t + \sum_{j=1}^{M}\sum_{\zeta=0}^{L} \omega^2_{ij\zeta}(\hat{\sigma}^2_{jt-\zeta} - \tilde{\sigma}^2_{t-\zeta}) + \sigma^2_{u_{it}} . \tag{6}$$

Where $\sigma^2_{u_{it}} = Var(u_{it})$ the regressions residual variance of APT with is lagged of stock return $i$ at the time $t$. Based on the assumptions in equation (6), covariance between stock $i$ with stock $i'$, which are stated to $\sigma_{ii'}$ be estimated with equation

$$\sigma_{ii't} = Cov(r_{it}, r_{i't}) = \sum_{j=1}^{M}\sum_{\zeta=0}^{L} \omega_{ij\zeta}\omega_{i'j\zeta}(\hat{\sigma}^2_{jt-\zeta} - \tilde{\sigma}^2_{t-\zeta}) ;\ i \neq i' . \tag{7}$$

Estimator mean, variance and covariance of stock return $i$ ( $i = 1,...,N$ and $N$ the number of stock that were analyzed), at the time $t$ ( $t = 1,...,T$ and $T$ the period of data observation), then used for the following portfolio formation.

### 2.4 Asset Allocation Based on Mean-VaR

Let $\ddot{r}_t$ denote the return of portfolio at the time $t$, and $w_i$ ( $i = 1,...,N$ ) weight of stock $i$. Return of portfolio $\ddot{r}_t$ can be determined using the equation [7; 13]:

$$\ddot{r}_t = \sum_{i=1}^{N} w_i r_{it} \text{ ; Terms } \sum_{i=1}^{N} w_i = 1 \text{ and } 0 < w_i < 1 \ ( i = 1,...,N ). \tag{9}$$

Suppose $\boldsymbol{\mu}^T = (\mu_{1t} \ ... \ \mu_{it})$ , $i = 1,...,N$ is the mean vector, and $\mathbf{w}^T = (w_1 \ ... \ w_N)$ the weight vector of portfolio. From equation (9), the weight $\mathbf{w}^T$ follows the property $\mathbf{e}^T\mathbf{w} = 1$ , where $\mathbf{e} = (1 \ ... \ 1)^T$ . The mean of portfolio return $\ddot{\mu}_t$ can be estimated using the equation:

$$\ddot{\mu}_t = \sum_{i=1}^{N} w_i \mu_{it} = \boldsymbol{\mu}^T \mathbf{w} . \tag{10}$$

The variance of portfolio return $\ddot{\sigma}_t^2$ can be estimated using the equation:

$$\ddot{\sigma}_t^2 = \sum_{i=1}^{N} w_i^2 \sigma_{it}^2 + \sum_{i=1}^{N}\sum_{i'}^{N} w_i w_{i'} \sigma_{ii'} = \mathbf{w}^T \mathbf{\Sigma} \mathbf{w};\ i \neq i'. \tag{11}$$

where $\sigma_{ii'} = Cov(r_{it}, r_{i't})$ denotes the covariance between stock $i$ and stock $i'$ [17].

*Value-at-Risk* (*VaR*) of an investment portfolio based on standard normal distribution approach is calculated using the equation [13; 19]:

$$VaR_t = -W_0(\ddot{\mu}_t + z_c \ddot{\sigma}_t) = -W_0\{\mathbf{w}^T \boldsymbol{\mu} + z_c (\mathbf{w}^T \mathbf{\Sigma} \mathbf{w})^{1/2}\}. \tag{12}$$

where $W_0$ the number of fund is allocated in the portfolio and $z_c$ is the percentile of standard normal distribution at the significance level $c$. When it is assumed $W_0 = 1$ unit, the equation (12) becomes:

$$VaR_t = -(\ddot{\mu}_t + z_c \ddot{\sigma}_t) = -\{\mathbf{w}^T \boldsymbol{\mu} + z_c (\mathbf{w}^T \mathbf{\Sigma} \mathbf{w})^{1/2}\}. \tag{13}$$

*A portfolio* $\mathbf{w}^*$ *is called* (*mean-VaR*) *efficient if there is no other portfolio* $\mathbf{w}$ *with* $\ddot{\mu}_t \geq \ddot{\mu}_t^*$ *and* $VaR_t < VaR_t^*$ [13]. To obtain the efficient portfolio, we used the objective function, to maximize $\{2\tau\ddot{\mu}_t - VaR_t\}$, $\tau \geq 0$ where $\tau$ denotes the investor risk tolerance factor. For the investor with the risk tolerance $\tau \geq 0$ therefore we must solve an optimization problem [13; 19]:

$$\begin{gathered}\text{Maximize } \{2\tau\boldsymbol{\mu}^T \mathbf{w} + \boldsymbol{\mu}^T \mathbf{w} + z_c (\mathbf{w}^T \mathbf{\Sigma} \mathbf{w})^{1/2}\} \\ \text{such that } \mathbf{e}^T \mathbf{w} = 1\end{gathered}. \tag{14}$$

Equation (14) is a quadratic concave optimization problem. Its Lagrangian function can be written as $L(\mathbf{w},\lambda) = (2\tau+1)\boldsymbol{\mu}^T \mathbf{w} + z_c (\mathbf{w}^T \mathbf{\Sigma} \mathbf{w})^{1/2} + \lambda(\mathbf{e}^T \mathbf{w} - 1)$. Using the Kuhn-Tucker theorem, the optimal solution can be obtained using the first derivatives, as follows [5; 7; 19]:

$$\partial L / \partial \mathbf{w} = (2\tau+1)\boldsymbol{\mu} + z_c \mathbf{\Sigma}\mathbf{w} / (\mathbf{w}^T \mathbf{\Sigma} \mathbf{w})^{1/2} + \lambda \mathbf{e} = 0 \text{ and } \partial L / \partial \lambda = \mathbf{e}^T \mathbf{w} - 1 = 0. \tag{15}$$

Solving the equation (15) as the function of $\lambda$, we obtain the quadratic equation in $\lambda$ as $(\mathbf{e}^T \mathbf{\Sigma}^{-1}\mathbf{e})\lambda^2 + (2\tau+1)(\boldsymbol{\mu}^T \mathbf{\Sigma}^{-1}\mathbf{e} + \mathbf{e}^T \mathbf{\Sigma}^{-1}\boldsymbol{\mu})\lambda + \{(2\tau+1)^2(\boldsymbol{\mu}^T \mathbf{\Sigma}^{-1}\boldsymbol{\mu}) - z_c^2\} = 0$. Let $A = \mathbf{e}^T \mathbf{\Sigma}^{-1}\mathbf{e}$, $B = (2\tau+1)(\boldsymbol{\mu}^T \mathbf{\Sigma}^{-1}\mathbf{e} + \mathbf{e}^T \mathbf{\Sigma}^{-1}\boldsymbol{\mu})$ and $C = (2\tau+1)^2(\boldsymbol{\mu}^T \mathbf{\Sigma}^{-1}\boldsymbol{\mu}) - z_c^2$. The roots of quadratic equations can be calculated using the ABC formula as [19]:

$$\lambda = \{-B + (B^2 - 4AC)^{1/2}\} / 2A;\ \lambda \geq 0. \tag{16}$$

For $\tau \geq 0$, we obtain the weight vector $\mathbf{w}$ as

$$\mathbf{w} = \frac{(2\tau+1)\mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{\Sigma}^{-1}\mathbf{e}}{(2\tau+1)\mathbf{e}^T \mathbf{\Sigma}^{-1}\boldsymbol{\mu} + \lambda\mathbf{e}^T \mathbf{\Sigma}^{-1}\mathbf{e}}. \tag{17}$$

By substituting the vector $\mathbf{w}$ into the equation (10) we obtain the mean value of portfolio return. When vectors $\mathbf{w}$ are substituted into the equation (13), we obtain the value of the investment portfolio risk level $VaR_t$. The sets of point pairs ($\ddot{\mu}_t, VaR_t$) form a graph of efficient frontier. Among the efficient frontier, there are optimum portfolios, which have the largest ratio $\ddot{\mu}_t / VaR_t$ [19].

## 3. Results and Analysis

### 3.1 The Data

For empirical study, we analyze the following stocks: PT. Astra International Industry (ASII), PT. Turba Alam Manunggal Engineering (TRUB), PT. Bank Central Asia (BBCA), PT. Bank Rakyat Indonesia (BBRI), PT. HM. Sampoerna (HMSP), and PT. Telekomunikasi Indonesia (TLKM), and are denoted as $S_1$ until $S_5$. As the factors index, we use Composite Stock Price Index (IHSG), the rate of inflation, exchange rate of the rupiah against the euro, the rupiah against the U.S. dollar, and the rupiah against the yen, and are denoted as $F_1$ until $F_5$. For the risk-free asset data, we use a government bond price. The data are obtained from http://www.finance.go.id//. The period of observation is January 2, 2006 until December 30, 2010. The empirical analysis is done using the software's: MS Excel 2007, Eviews 4, Maple 9.5 and R.

### 3.2 Empirical Results

In this study, the factors index used are $F_1$ until $F_5$, as described above. We first calculate the returns of each factor index, then identify the existence of the effects of long memory in the returns, and finally estimated the mean and volatility models of the returns.

***Identification of the long memory effects***. To identify the effect of long memory, we estimate the parameters of fractional difference $d_j$ ($j = 1,...,5$) as in equation (1). The estimation is performed using the rescale range method (R/S) or Geweke and Porter-Hudak (GPH) method. The results are summarized in Table-1.

**Tabel-1.** The Identification of the Long Memory Effects

| Stock | $\hat{d}_j$ | $\sigma_{d_j}$ | Intervals Confidence | $z_j$ | Effect of Long Memory |
|---|---|---|---|---|---|
| $F_1$ | 0.361 | 0.1462 | $0.075 < d_1 < 0.648$ | 5.86 | Significant |
| $F_2$ | 0.263 | 0.1323 | $0.004 < d_2 < 0.522$ | 2.57 | Significant |
| $F_3$ | 0.635 | 0.5853 | $-0.512 < d_3 < 1.782$ | 1.12 | Not Significant |
| $F_4$ | 0.098 | 0.1731 | $-0.241 < d_4 < 0.437$ | 2.62 | Significant |
| $F_5$ | 0.518 | 0.6312 | $-0.719 < d_5 < 1.755$ | 1.51 | Not Significant |

To ensure the existence of long memory patterns, we test the hypothesis $H_0 : d_j = 0$ against, $H_1 : d_j \neq 0$, $j = 1,...,5$. Statistic values calculated $z_j$ ($j = 1,...,5$) are given in Table-1, while for the level of significance $c = 0.95$, from the standard normal distribution table values, we obtain $Z_{0.95/2} = 1.96$. Because of the values $z_1$, $z_2$ and $z_4$ are larger than the value $Z_{0.95/2}$, it is concluded that the test results are significant, the returns of factor index data, $F_1$, $F_2$ and $F_4$ have long memory effects. However, in $F_3$ and $F_5$ there are no long memory effects.

In the next step, we identify and estimate the best mean and volatility models to difference fractional $\hat{d}_j$ of the returns data $F_1$, $F_2$ and $F_4$, where for F3 and F5, the analysis is applied directly to the returns data.

***Identification and estimation of mean models***. Identification of the mean models is done using the sample *autocorrelation function* (ACF) and *partial autocorrelation function* (PACF). Based on the patterns of ACF and PACF of each factor index returns (or the fractional differenced data), we obtain the best models for F1 until F5, which also passed the standard diagnostic check. The results are summarized in Table 2.

***Identification and estimation of volatility models***. We further identify and estimate of volatility model using *generalized autoregressive conditional heterscedasticity* (GARCH) models. Based on the correlogram of quadratic residual $a_{jt}^2$, we select the plausible volatility model for the data. Estimation of volatility models of each factors index return is done simultaneously with the mean models. The results, obtained for the best model which is also passed the diagnostic checks, are given in Tabel-2.

**Table-2.** The Estimation Results of Mean and Volatility Model of Factor Index Returns

| Factor ($F_j$) | Model | Mean and Volatility Equations |
|---|---|---|
| $F_1$ | ARFIMA(1, $\hat{d}_1$,0)-GARCH(1,1) | $\hat{r}_{1t} = 0.111341\hat{r}_{1t-1} + a_{1t}$<br>$\hat{\sigma}^2_{1t} = 0.00000866 + 0.137021 a^2_{1t-1} - 0.834528\hat{\sigma}^2_{1t-1} + \varepsilon_{1t}$ |
| $F_2$ | ARFIMA(1, $\hat{d}_2$,1)-GARCH(1,1) | $\hat{r}_{2t} = 0.993306\hat{r}_{2t-1} - 0.990698 a_{2t-1} + a_{2t}$<br>$\hat{\sigma}^2_{2t} = 1.016328 + 0.447513 a^2_{2t-1} - 0.043462\hat{\sigma}^2_{2t-1} + \varepsilon_{2t}$ |
| $F_3$ | AR(1)-GARCH(1,2) | $\hat{r}_{3t} = -0.070772\hat{r}_{3t-1} + a_{3t}$<br>$\hat{\sigma}^2_{3t} = 0.000000853 + 0.140811 a^2_{3t-1} + 0.300641\hat{\sigma}^2_{3t-1} + 0.563666\hat{\sigma}^2_{3t-2} + \varepsilon_{3t}$ |
| $F_4$ | ARFIMA(1, $\hat{d}_4$,0)-GARCH(2,1) | $\hat{r}_{4t} = -0.078681\hat{r}_{4t-1} + a_{4t}$<br>$\hat{\sigma}^2_{4t} = 0.00000837 + 0.38691 a^2_{4t-1} + 0.478577 a^2_{4t-2} + 0.372516\hat{\sigma}^2_{4t-1} + \varepsilon_{4t}$ |
| $F_5$ | AR(1)-TGARCH(2,1) | $\hat{r}_{5t} = -0.094107\hat{r}_{5t-1} + a_{5t}$<br>$\hat{\sigma}^2_{5t} = 0.000000467 + 0.280911 a^2_{5t-1} + 0.234836 a^2_{5t-2} + 0.951865\hat{\sigma}^2_{5t-1} + \varepsilon_{5t}$ |

***Estimated regression model of APT with lagged***. In this section estimation of APT model with lagged, conducted by estimating regression models of each of the five stock return data, against the data of return of the five-factor index. Estimation made refers to the equation (4), helped by Eviews 4 software. The return of risk-free asset data (bond) is relatively constant, therefore, taken the mean size $\tilde{\mu}_t = 0.026462$ and variance $\tilde{\sigma}^2_t = 0$. To simplify the writing, for example $\varsigma_{it} = r_{it} - 0.026462$, $i = 1,...,5$ and the risk premium of factor index with the lagged $I_{jt-\zeta} = \hat{r}_{jt-\zeta} - 0.026462$ ($j = 1,...,5$ and $\zeta = 0,1,...,L$, where $L$ length of is lagged). The results of APT regression with lagged estimates given in Table-3. Numbers written in parentheses under the regression coefficients are t-Statistic. The values of the coefficient of determination is given in the column $R_i^2$, whereas the values Darbin-Watson Statistic are given in the column Stat-$DW$.

**Table-3.** Regression Model Estimation Results of APT with lagged

| Samah ($S_i$) | Regression Model | $R_i^2$ | Stat-$DW$ |
|---|---|---|---|
| $S_1$ | $\underset{Stat-t}{\varsigma_{1t}} = \underset{(3.74)}{0.0251} + \underset{(17.91)}{1.4723} I_{1t} + \underset{(3.74)}{0.2994} I_{1t-1} + \underset{(1.65)}{0.1205} I_{1t-2} - \underset{(-2.43)}{0.0005} I_{2t} - \underset{(-3.42)}{0.5251} I_{3t} - \underset{(-2.19)}{0.1840} I_{3t-1}$<br>$+ \underset{(2.67)}{0.6843} I_{4t} + \underset{(2.45)}{0.1151} I_{4t-1} - \underset{(-2.28)}{0.0535} I_{5t} - \underset{(-2.09)}{0.0171} I_{5t-1} + u_{1t}$ | 58.75% | 1.944 |
| $S_2$ | $\underset{Stat-t}{\varsigma_{2t}} = \underset{(1.66)}{0.0040} + \underset{(14.11)}{0.9429} I_{1t} - \underset{(-1.83)}{0.0009} I_{2t} - \underset{(-2.53)}{0.0006} I_{2t-1} - \underset{(-2.31)}{0.0003} I_{2t-2} + \underset{(2.35)}{0.0442} I_{3t} + \underset{(2.25)}{0.1466} I_{3t-1}$<br>$+ \underset{(3.25)}{0.0239} I_{3t-2} + \underset{(2.46)}{0.0432} I_{3t-3} + \underset{(2.58)}{0.1236} I_{4t} - \underset{(-1.66)}{0.2557} I_{5t} - \underset{(-2.12)}{0.0110} I_{5t-1} + u_{2t}$ | 47.79% | 2.081 |
| $S_3$ | $\underset{Stat-t}{\varsigma_{3t}} = \underset{(2.52)}{0.0091} + \underset{(19.10)}{1.4491} I_{1t} + \underset{(1.83)}{0.1313} I_{1t-1} - \underset{(-1.90)}{0.0011} I_{2t} - \underset{(-1.98)}{0.0012} I_{2t-1} - \underset{(-2.55)}{0.2223} I_{3t} - \underset{(-2.21)}{0.0515} I_{4t}$<br>$- \underset{(-2.06)}{0.0072} I_{4t-1} - \underset{(-2.11)}{0.0190} I_{5t} + u_{3t}$ | 60.56% | 1.963 |
| $S_4$ | $\underset{Stat-t}{\varsigma_{4t}} = \underset{(2.26)}{0.0022} + \underset{(4.52)}{0.3894} I_{1t} + \underset{(1.65)}{0.0978} I_{1t-1} + \underset{(2.03)}{0.0822} I_{1t-2} - \underset{(-2.22)}{0.0014} I_{2t} - \underset{(-1.66)}{0.1061} I_{3t} - \underset{(-1.70)}{0.2743} I_{3t-1}$<br>$+ \underset{(2.51)}{0.1369} I_{4t} + \underset{(2.36)}{0.0966} I_{4t-1} + \underset{(1.91)}{0.2395} I_{4t-2} + \underset{(2.00)}{0.1977} I_{5t} + \underset{(2.09)}{0.2174} I_{5t-1} + u_{4t}$ | 48.50% | 2.553 |
| $S_5$ | $\underset{Stat-t}{\varsigma_{5t}} = \underset{(-2.14)}{-0.0007} + \underset{(14.92)}{0.9196} I_{1t} - \underset{(-1.80)}{0.0007} I_{2t} - \underset{(-2.31)}{0.2695} I_{3t} + \underset{(2.04)}{0.0075} I_{4t} + \underset{(2.59)}{0.0840} I_{5t}$<br>$+ \underset{(2.10)}{0.007} I_{5t-1} + \underset{(1.66)}{0.1067} I_{5t-2} + \underset{(2.54)}{0.0997} I_{5t-3} + u_{5t}$ | 46.66% | 1.905 |

As presented by Blume in 1971, the ability of the index factors to explain changes in individual stock returns ranging between 25% -51% rate coefficient of determination [7]. Looking at the results in Table-3, it appears that the coefficient of determination $R_i^2$ of regression models of each stock worth nearly 51%. So for a regression of return or risk premium that has a coefficient of determination close to 51% was considered strong enough, as long as other diagnistik test statistic is significant [10]. Based DW statistic whose value is also relatively small, showing the five regression models in Table-3 is significant. It can also be shown the residuals $u_{it}$ of each regression model are normally distributed, with zero mean and variance $\sigma_{u_{it}}^2$ . Estimator value $\sigma_{u_{it}}^2$ of each regression equations are given in Table-4. Regression model is then used to estimate the mean and variance of stock return.

***The estimated mean and variance values of stocks return.*** The values of constants, coefficient of parameters and regression of residual variance $\sigma_{u_{it}}^2$ in Table-3, and the mean value estimator $\hat{\mu}_{jT} = \hat{r}_{jT}(1)$ and variance $\hat{\sigma}_{jT}^2 = \hat{\sigma}_{jT}^2(1)$, then used to estimate the mean and variance values of stock return $S_1$ until $S_5$. The mean value estimated using equation (6), while the variance value is estimated based on equation (7). The estimation results are given in Table-4.

**Table-4.** The Estimation Results of Mean and Variance of Stocks Return

| **Stocks** **($S_i$)** | $\sigma_{u_{it}}^2$ | $\mu_{it}$ | $\sigma_{it}^2$ |
|---|---|---|---|
| $S_1$ | 0.000788 | 0.001876 | 0.001278 |
| $S_2$ | 0.000536 | 0.002328 | 0.000732 |
| $S_3$ | 0.000705 | 0.001835 | 0.001132 |
| $S_4$ | 0.000852 | 0.000231 | 0.000929 |
| $S_5$ | 0.000456 | 0.000797 | 0.000634 |

In this study assumed that the covariance between stock return $i$ and $i'$ at the time $t$, $\sigma_{ii't} = 0$ ($i,i' = 1,...,5$ and $i \neq i'$), because its values are also very small close to zero. This means that between stock return $i$ and $i'$ at the time $t$ where the cross correlation does not occur. The values of the mean estimator $\mu_{it}$ and the variance $\sigma_{it}^2$ will be used for the following portfolio optimization process.

***Portfolio optimization.*** Portfolio optimization in this case is based on the *mean-VaR* model. To solve optimization problem is done by using the software of *Maple 9.5.* First, the values of the average estimator in Table-4 column $\mu_{it}$ is used to form the vector mean as $\mathbf{\mu}^T = (0.001867 \;\; 0.002328 \;\; 0.001835 \;\; 0.000231 \;\; 0.000797)$. Second, referring to the number of shares to be analyzed as much as five, mean vector defined identity is $\mathbf{e}^T = (1\ 1\ 1\ 1\ 1)$. Third, the values of the variance estimator in Table-4 a column $\sigma_{it}^2$ is used to form the covariance matrix $\mathbf{\Sigma}$ . Then determined the inverse of the matrix $\mathbf{\Sigma}$, ie $\mathbf{\Sigma}^{-1}$, as follows:

$$\mathbf{\Sigma} = \begin{pmatrix} 0.00128 & 0 & 0 & 0 & 0 \\ 0 & 0.00073 & 0 & 0 & 0 \\ 0 & 0 & 0.00113 & 0 & 0 \\ 0 & 0 & 0 & 0.00093 & 0 \\ 0 & 0 & 0 & 0 & 0.000634 \end{pmatrix} \text{ and } \mathbf{\Sigma}^{-1} = \begin{pmatrix} 782.47 & 0 & 0 & 0 & 0 \\ 0 & 1366.12 & 0 & 0 & 0 \\ 0 & 0 & 883.39 & 0 & 0 \\ 0 & 0 & 0 & 1076.43 & 0 \\ 0 & 0 & 0 & 0 & 1577.29 \end{pmatrix}$$

Risk tolerance $\tau$ in this study are determined by simulation. Tolerance values together $\boldsymbol{\mu}^T$ and $\mathbf{e}^T$ vectors and $\boldsymbol{\Sigma}^{-1}$matrix, when substituted into equation (15) will be obtained multiplier value as $\lambda$. Where then substituted into equation (16) will be obtained by portfolio weight vector $\mathbf{w}$. Weight vector $\mathbf{w}$, then used to calculate the mean value of portfolios using equation (10), and calculate the risk level of VaR using equation (13). For the values of risk tolerance $0 \le \tau \le 7.377$ some calculation results are given in Table-5.

**Table-5.** Risk Tolerance, Weight, Mean and Portfolio Risk

| $\tau$ | Weight | | | | | $\ddot{\mu}_t$ | $VaR_t$ | $\frac{\ddot{\mu}_t}{VaR_t}$ |
|---|---|---|---|---|---|---|---|---|
| | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | | | |
| 0.00 | 0.14083 | 0.25086 | 0.15870 | 0.17945 | 0.27015 | 0.001396 | 0.0204341 | 0.06832617 |
| 0.50 | 0.14405 | 0.26148 | 0.16205 | 0.16954 | 0.26287 | 0.001425 | 0.0204485 | 0.06968778 |
| 1.00 | 0.14730 | 0.27220 | 0.16542 | 0.15956 | 0.25552 | 0.001454 | 0.0204922 | 0.07095751 |
| 1.50 | 0.15059 | 0.28304 | 0.16883 | 0.14945 | 0.24809 | 0.001483 | 0.0205657 | 0.07213373 |
| 2.00 | 0.15393 | 0.29406 | 0.17230 | 0.13918 | 0.24053 | 0.001513 | 0.0206704 | 0.07321444 |
| 2.50 | 0.15734 | 0.30531 | 0.17584 | 0.12869 | 0.23282 | 0.001544 | 0.0208077 | 0.07419726 |
| 3.00 | 0.16084 | 0.31683 | 0.17947 | 0.11795 | 0.22491 | 0.001575 | 0.0209798 | 0.07507931 |
| 3.50 | 0.16444 | 0.32871 | 0.18321 | 0.10688 | 0.21677 | 0.001607 | 0.0211892 | 0.07585733 |
| 4.00 | 0.16817 | 0.34100 | 0.18707 | 0.09542 | 0.20834 | 0.001641 | 0.0214394 | 0.07652736 |
| 4.50 | 0.17204 | 0.35379 | 0.19110 | 0.08349 | 0.19957 | 0.001702 | 0.0234100 | 0.07271306 |
| 5.00 | 0.17610 | 0.36718 | 0.19531 | 0.07101 | 0.19039 | 0.001712 | 0.0220795 | 0.07752462 |
| 5.50 | 0.18038 | 0.38127 | 0.19975 | 0.05787 | 0.18073 | 0.001750 | 0.0224812 | 0.07784036 |
| 6.00 | 0.18491 | 0.39622 | 0.20446 | 0.43936 | 0.17048 | 0.001790 | 0.0229477 | 0.07802487 |
| **6.404** | **0.18880** | **0.40903** | **0.20849** | **0.03199** | **0.16169** | **0.001825** | **0.0233789** | **0.07807227** |
| 6.50 | 0.18975 | 0.41219 | 0.20948 | 0.29050 | 0.15953 | 0.001834 | 0.0234893 | 0.07806955 |
| 7.00 | 0.19497 | 0.42939 | 0.21490 | 0.01301 | 0.14773 | 0.001880 | 0.0241196 | 0.07796420 |
| 7.377 | 0.19920 | 0.44335 | 0.21929 | 0.000003 | 0.13816 | 0.001918 | 0.0246640 | 0.07777817 |

For values of risk tolerance $\tau > 7.377$ is no longer feasible, because the portfolio weight does not qualify.

A collection of points ( $\ddot{\mu}_t, VaR_t$ ) to form the surface efficiently (efficient frontier), as shown in Figure-1. The ratio between $\ddot{\mu}_t$ and $VaR_t$ values are given in Table-5 the last column. The values of these ratios can also be expressed as a graph given in Figure-2.

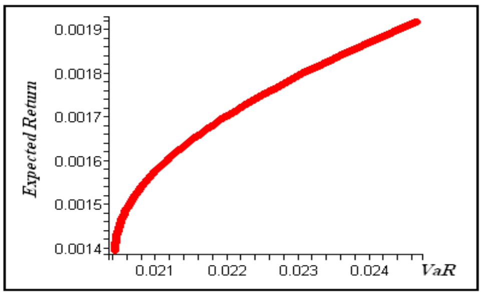


**Figure -1.** Efficient Frontier of Portfolio

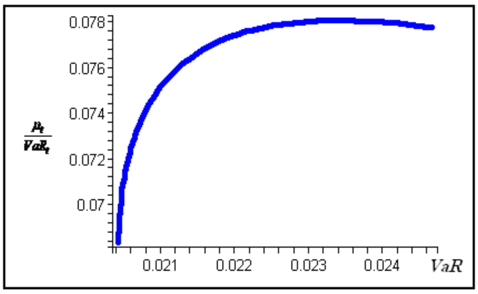


**Figure-2.** Mean and VaR Ratio of Portfolio

### 3.3 Discussion

Based on the calculation results given in Table-5, can be seen that with the risk tolerance of 0.00 obtained VaR portfolio composition that produces a minimum, that is equal to 0.020434 with the expected return of portfolio amounted to 0.001396. The amount of risk tolerance can still be improved but with the condition that the resulting values of weighted portfolio of real $0 < w_i < 1$ ($i = 1,...,5$) and qualify $\sum_{i=1}^{5} w_i = 1$. In this case the value will be at most risk tolerance as $\tau = 7.377$. Where the resulting composition of the portfolio with the highest expected return of portfolio that is equal to

0.0019183 with VaR at 0.024664. Any increase in the value of risk tolerance will cause the increase in expected return portfolio which is also accompanied by an increase in the Value-at-Risk portfolio.

Efficient portfolios lie along the line with risk tolerance of $0 \le \tau \le 7.377$, as given by the graph of the surface efficiently (efficient frontier) in Figure-1. The curve is the collection of pairs of points ($\hat{\mu}_t, VaR_t$) that can be chosen by the investor to invest in a portfolio of $S_1$, $S_2$, $S_3$, $S_4$ and $S_5$. Of course, each investor should choose one based on preference or risk tolerance level that is believed. For the avoidance of risk investors will usually sets the risk tolerance is small, while for investors challenger will risk taking a big risk tolerance. Large-size specified risk tolerance, of course, will affect the large-size of portfolio expected return obtained.

Having obtained a series of efficient portfolios, the next step is determining the optimum portfolio composition. Any investor would want the optimum investment portfolio, is a portfolio that minimizes risk and maximizes the expected return. Selection of the optimum portfolio can be determined based on the composition of the efficient portfolios that generate expected return and the Value-at-Risk of portfolio with the largest ratio.

Based on the calculation results given in the last column of Table-5, shows that the ratio of expected return and VaR the largest portfolio is 0.07807227, or obtained when risk tolerance reaches $\tau$ =6404. The ratio of expected return and VaR continues to increase at intervals of risk tolerance $0 \le \tau \le 6.404$ and decreased in the interval $6.404 \le \tau \le 7.377$. Up-and-downs of these ratios can be seen graph given in Figure-2. Based on the values in Table-5, obtained the results that based on the model of asset allocation mean-VaR, the optimum portfolio composition prepared from the stocks $S_1$, $S_2$, $S_3$, $S_4$ and $S_5$ the portfolio composition with the weight vector $\mathbf{w}^T$ = (0.1888 0.4090 0.2085 0.0320 0.1617). Where is the optimum portfolio composition to generate the expected return of $\hat{\mu}_t$ =0.001825 with Value-at-Risk of $VaR_t$ = 0.0233789. These values of $\hat{\mu}_t$ and $VaR_t$ can be seen in Table-5 line in bold.

## 4. Conclusion

Mathematical of *Arbitrage Pricing Theory* (APT) model can be expanded into APT with lagged. Based on it the estimators of mean, variance and covariance of stock return are formulated. The empirical research conducted on five stocks $S_1$ until $S_5$, and five index factor of $F_1$ until $F_5$. Where return of the index factors in the APT with lagged analyzed using ARFIMA-GARCH model approach. Based on the analysis that the three index return factor, namely $F_1$, $F_2$ and $F_4$ there are significant effects of long memory. Parameters estimator of mean, variance and covariance, then used for the analysis of asset allocation problem based on the mean-VaR model. Based on the results of optimization, efficient portfolios are formed for the values of risk tolerance $0 \le \tau \le 7.377$. An optimum occurs in the value of portfolio risk tolerance $\tau = 6.404$. Optimum portfolio has composition allocation weight of $\mathbf{w}^T = (0.1888 \;\; 0.4090 \;\; 0.2085 \;\; 0.0320 \;\; 0.1617)$, with a mean portfolio return of $\hat{\mu}_t$ =0.001825 and Value-at-Risk of $VaR_t$ =0.0233789.

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